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Wavefront sets of solutions to linearised inverse scattering problems

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[redacted]

In [10] precise answers to these questions are given using the notion of a generalised Radon transform [23]. In particular the answer is affirmative to the first question.

The purpose of this short paper is to reformulate the results of [10] in terms of wavefront sets. This provides a natural framework for formulating the inverse problem in general as one of determination of wavefront sets of unknown parameters of PDEs, which is a meaningful question to ask from the point of view of applications.

Such a formulation is also meaningful from the pure mathematical point of view. In other words, the wavefront set of a function seems to be a natural notion to use in problems of reconstructing discontinuities. Recall that the concept of the wavefront set is

satisfying the eikonal equation

$$|\nabla_x \varphi(x, \xi)|^2 = n_0^2(x) \tag{3.2}$$

for $x \in X$, and $\xi \in \partial X$, such that

$$\varphi(x, \xi) \to 0 \qquad \text{as } x \to \xi. \tag{3.3}$$

We also assume that $\varphi(x, \xi)$ satisfies conditions (2.1), (2.2). For example if $n_0(x)$ is constant, say $n_0(x) = 1$—the constant background model—we have states that the locations of discontinuities together with their infinitesimal directions can be (at least partially) recovered. In fact the migration schemes described in [13–21] all have [redacted]

Appendix

Here we state the basic facts about pseudodifferential operators. Let X be an open set of R^n.

Definition (cf [7] vol. 1, p 13). The space of symbols of degree m, denoted by

$$S^m(X \times X \times R^n \backslash \{0\})$$

consists of all complex-valued functions

$$C(x, y, \theta) \in C^\infty(X \times X \times R^n \backslash \{0\})$$

[redacted]

where

$$p = \nabla_x \Phi(x, x, \theta).$$

Since the determinant factor is non-zero and positive homogeneous degree 0 in p, we conclude that T is elliptic at (x_0, p_0) if

$$|C(x, x, \theta)| \geqslant d|\theta|^m$$

for all (x, θ) in some conic neighbourhood of (x_0, θ_0) where

$$p_0 = \nabla_x \Phi(x_0, x_0, \theta_0)$$

and d is some positive constant. From this we get the following corollary.

Corollary A.2. Let $Tf = g$ be as in proposition A.1, then

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