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The Location of the Poor in a Metropolitan Area: Positive and Normative Analysis

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ABSTRACT

We seek to explain the stylized fact that poor households form the majority in the inner city of most American metropolitan areas. Using numerical simulations, we show that (1) typically there exist two equilibria: one in which the poor form the majority in the inner city and the other in which the rich form the majority; (2) when the metropolitan population is small, rich households "jump to the suburb" to obtain their desired public service, and this causes the growth path to select the equilibrium in which poor households are the inner city's majority; (3) whether having the poor form the inner city's majority is desirable depends on the metropolitan population and on whether efficiency or equity is the normative criterion.

JEL Classification: H73, R12, R14.

In contrast to Tiebout's model, Alonso (1964) Mills (1967) and Muth (1969) downplay the fiscal difference between communities and instead stress spatial aspects. They consider a metropolitan area to be an "as if" single monocentric city to whose center households commute. Land prices decrease as the location moves away from the center, reflecting the increase in the commuting cost. Income sorting still occurs, but it now arises from the interaction of commuting costs and land demand. If land demand is sufficiently income elastic, the saving achieved by the purchase of land further from the city's center is greater for the rich households and compensates them for the associated increase in commuting cost.[2] In this case rich households live on the outside of the metropolitan area and poor households live in the inner city.   Conversely, if land demand is unresponsive to income changes and commuting costs increase with income, the greater commuting benefit to rich households of living closer to center causes them to outbid poor households for the locations closer to the city's center. Summarizing, whether rich households sort themselves on the outside or on the inside of the metropolitan area depends on whether the income elasticity of land demand is greater or smaller than the income elasticity of the cost of commuting.

Wheaton (1977) determines that the income elasticity of land demand is statistically indistinguishable from the income elasticity of the cost of commuting, so that the Alonso-Mills-Muth model is unable to predict whether it is the poor or the rich who live in the inner city. Glaeser, Kahn and Rappaport (2000) find evidence that the income elasticity of land demand is quite small, so that the monocentric city model predicts that it is the rich who live in the inner city. Empirically, therefore, this type of sorting cannot play a major role in explaining the centralization of the poor. Instead, Glaeser et al. suggest that the better access to public transportation - used by the poor to commute to the city's center - is the primary reason why it is the poor who congregate in the inner cities.

Although neither the Tiebout model nor the Alonso-Mills-Muth model on their own can explain the concentration of the poor in the inner cities, we find that a model with fiscal differences and commuting costs can explain it. In our model, a circular inner city has an exogenous boundary and is surrounded by a suburb. The public service in each community is determined by voting and all households must commute to the central business district which is located at the center of the inner city. The model has two income-classes. Rich households have a higher demand for the public service so that *ceteris paribus* different income groups prefer to live in different communities. Rich households have higher commuting costs per mile than poor households and land demand is relatively income inelastic. *Ceteris paribus*, therefore, rich households outbid poor households for land nearer the city's center. In the spirit of the indeterminacy of Teibout's model, we find multiple equilibria over a range of metropolitan populations. In one equilibrium, it is the poor households who form the majority in the inner city, voting low public services in that city; in the second equilibrium, it is the rich households who form the majority in the inner city, voting high public services there.

To determine which equilibria is likely to be selected, we consider the city's growth to be approximated by the comparative statics of an increase in the metropolitan population in the presence of a fixed city boundary. When the population is small, all households live in the inner city; there is a majority of poor households and the public service is low. As the population increases, city rents increase and the edge of urban development moves towards the city boundary. While there is still some undeveloped land in the city, some rich households "jump to the suburb" to form a new community with a high public service. This establishes rich households as the majority in the suburb. As the metropolitan area's population further expands, the configuration of households is maintained: poor households congregate in the inner city and rich households congregate in the suburb providing a high public service. At very high population sizes, high suburban rents plus commuting costs induce rich households to start to move back to the city - gentrification - and at sufficiently large populations the city changes again to have a majority of rich households.

What matters in our model is the difference in the public service level desired by poor and rich households. For expositional simplicity, we have considered a single public service which is a normal good; an example is public education. However, the qualitative description is unchanged if the public service is changed to be an inferior good; an example is public transportation. When the metropolitan area has a small population, all households live in the city and the poor households, forming the majority, vote high public transportation expenditures. As the metropolitan population expands, rich households "jump to the suburb" to escape the high

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of American metropolitan areas. What are the normative implications of this prediction? In particular: does the selected equilibrium match the preferred equilibrium? We find that the answer to this question depends on the normative criterion - efficiency or equity - and on the metropolitan population. In metropolitan areas with small populations, the equilibrium in which poor households form the inner city

metropolitan center, he spends no time commuting and his income is M. If he lives at distance d from the metropolitan center, his income is reduced by the opportunity cost of the commute. The time spent commuting is proportional to d and the opportunity cost of a unit of his time is proportional to M, so that his commuting cost is tMd. The household's exogenous demand for land area is a and the price of a unit of land at d is $r(d)$. The community provides the public service g. The public service shows constant returns to community size, and the cost of providing a unit of the public service to each resident is one unit of numeraire per resident [4] ; the public service is financed by community:

———— (1)

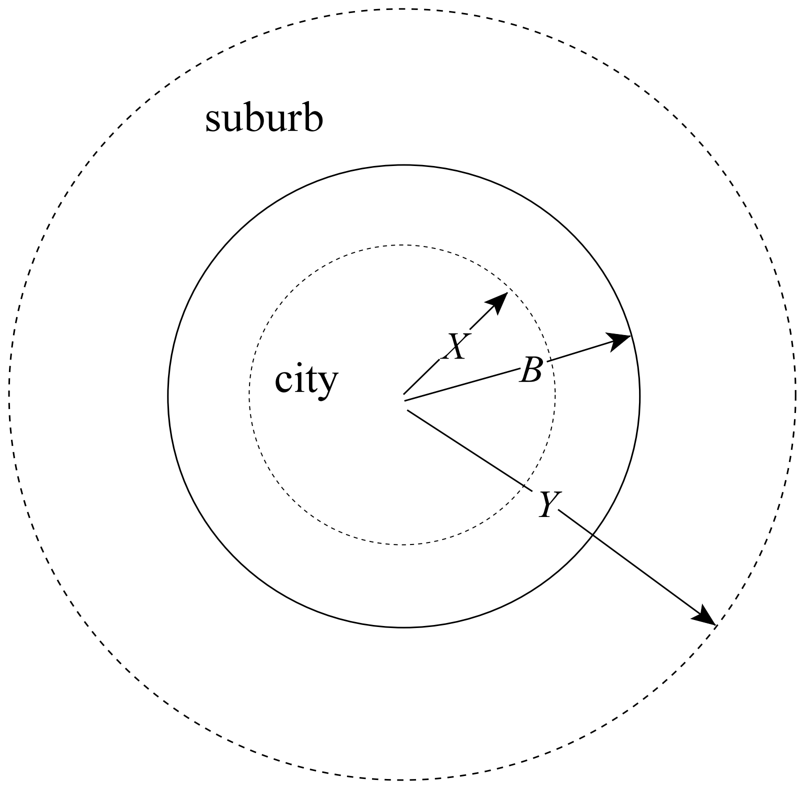

Figure 1: the metropolitan area

The metropolitan area is comprised of a circular inner city (henceforth termed “the city”) surrounded by a”s)”ub)urb. The jurisdictional bound2.7600 l cm.0800 11.2800 an area is cictiona 000 1.00000 etwe

Remembering that rich households live on the inside of the city and poor households live on the outside, denote the boundary between rich and poor households in the city as occurring at distance x from the center. If

$x = 0$: only poor households live in the city;

$0 < x < X$: rich and poor households live in the city;

$x = X$: only rich households live in the city

1. Rent continuity: rent is continuous in a community. If it were discontinuous, a household living on the side of relatively high rent could increase his utility by moving across the discontinuity to the side of low rent: his rent would change discontinuously but his commuting cost would change only marginally.[7]
2. No migration: no household can achieve higher utility by moving to another location. This implies that, if an income class resides in both communities, the rents are such that a household in that income class is indifferent between the communities. If an income class does not reside in a community, rents are such that a household in that income class cannot increase his utility by moving into the community.
3. Reservation land price: the reservation price of land is r_0. If a community contains no undeveloped land, the rent at the limit of development is at least r_0. If a community contains undeveloped land, the rent at the limit of development is r_0.
4. Determination of the public service level. The public service level in each community is determined by majority voting; households vote myopically, taking the rent schedules as given.[8]
5. Model closure. We assume that rent is paid to absentee landlords.[9]
6. The population in each community is considered to be a continuous variable.

There are many possible equilibrium configurations corresponding to which income class forms the majority in the city and whether the city includes one or both income classes, which income class forms the majority in the suburb and whether the suburb includes one or both income classes, and whether there is undeveloped land in the city.[10] In our simulations, we have set the numbers of poor and rich households to be equal (technically there is one additional poor household to ensure that the metropolitan area has an overall majority of poor households). With a sufficiently small metropolitan population, there may be only one equilibria in which all households live in the city: the potential cost of commuting from the suburb deters households from living in the suburb. At a larger metropolitan population, both jurisdictions are occupied. We now describe these latter equilibria. With the assumed equal numbers of poor and rich households, there are two configurations: Case 1 in which the city has a majority of poor households and the suburb has a majority of rich households, and Case 2 in which the majorities are reversed. We restrict attention to the equilibria found by our simulations, and consider Case 2(ove)Tj44.5200 0. 2(ove)Tm2j46.6gul8o411 of poor Td1ecwguld ric30.00 rgBT72s0000000 TD(of poor of r8r12BT7d1

households across the jurisdictional boundary. Rents paid by poor households in the city are still sufficiently low that poor households do not wish to migrate into the suburb.

As the metropolitan population increases further, the city fills with poor households, driving rich households out of the city into the suburb. At a critical population, the city is filled with all the poor households; if the metropolitan population increases further, the additional rich and poor households locate in the suburb, and Case 1.3 applies. Poor households live in both locations and the level of the city's rent schedule adjusts to ensure that poor households achieve the same utility in the city as in the suburb: *DEK* is the bid-rent curve of poor households in the suburb and *KH* is the rent premium a poor household is willing to pay to acquire its desired public service.

Case 2: rich households are majority in city, poor households are majority in suburb.household

3. SIMULATIONS

3.1 Analytical Framework

The utility function of a household of income M_i achieving consumption c and receiving public service g is specified as

$c \; + \; sign(\rho$

Parameter	Variable Name	Model Value	Societal Value
Poor households as fraction of population	2	0.5	0.5
Income of poor household ($ per year) [a]	M_1	15,000	16,523
Income of rich household ($ per year) [a]	M_2	45,000	46,725
Average lot size (acres) [b]		0.3333	0.3402
Lot size of poor household (acres) [c]	a_1	0.2833	
Lot size of rich household (acres) [c]	a_2	0.3833	
Commute time per mile as fraction of work day [d]	t	0.013	
Public service demand parameter [c]	*	1.4	
Public service demand parameter [c]	D	-1.0	
Public service demand parameter [c]	A	3.0	
Reservation rent [c]	r_0	0	

a. Societal figures are total 1990 money income at 25th and 75th percentile in metropolitan income distribution (*Money Income of Households, Families and Persons in the United States 1990*, Table 2) multiplied by *(1*-J*)* where J is average federal income tax rate for federal tax return with adjusted gross income equal to money income of respective household (*Individual Income Tax Returns* 1990). Model value represents earned income of a household with zero commuting costs.

b. Societal average lot size is metropolitan land area divided by societal metropolitan population. Metropolitan land area calculated as 1991 occupied housing units in all central cities multiplied by median lot size in central city plus same 1991 figure for suburban units (*American Housing Survey for the United States* 1991, Tables 8-3 and 9-3). Societal metropolitan population is calculated as 1991 number of owner-occupied units in all central cities plus suburbs (*American Housing Survey for the United States 1991*, Tables 8-3 and 9-3).

c. See text

d. Fraction of 8-hour workday spent commuting to metropolitan center if household lives one mile from city center. Figure is based on an average travel speed of 20 miles per hour and a round trip commute.

Table 1: Parameter and Population Values

The population and average lot size are chosen to be close to the values observed in the population. The lot size for poor and rich households is adjusted down and up from the average lot size, respectively, in order to be consistent with a 0.3 income elasticity of demand for land - an elasticity value which is consistent with the recent estimates by Glaeser et al. (2000).[12] The population and housing demand parameters imply that the area of developed land in the metropolitan area is 90,000 acres: this translates to a radius (*Y*) of 6.7 miles if the city has no vacant land.

The values for D and *

4. SUBURBANIZATION IN A GROWING CITY

Current metropolitan areas have grown out of much smaller cities. Does the likely growth path select an equilibrium with a majority of poor or rich households in the city? Our presumption is that marginal increases in the population are accommodated by changes in the boundary between the income groups and not by large population shifts between the communities. Put differently, once an equilibrium configuration of majorities is established, we presume that it is maintained as the population grows (provided the configuration continues to be an equilibrium).

Historically, when metropolitan populations were small compared to the size of the city, all households lived in the city and poor households formed the majority. As the metropolitan population grew, our model r 170000 0.0000 TD(sctcreases174spopulat00 TD53en the income grous poor hous

experienced by a poor and rich household moving to set up a new community in the suburb decreases. These utility changes are shown on Lines 12 and 13a of Table 2 on Page 25. At *N=1,000* and *N=20,000* the utility difference is large and negative for poor households (-896 and -620 ($ per year) respectively[19]); for rich households the utility change is large and negative at *N = 1,000*, is very small at *N=20,000*, is zero at *N = 22,461* (not shown) and is positive thereafter. Therefore, as the metropolitan population expands beyond 22,461, rich households have the incentive to "jump over" the undeveloped land to form a new community in the suburb with the form of Case 1.1. In contrast, the establishment of Case 2.1 requires that some poor households move to the suburb although this makes them worse off. We predict therefore that the growth path selects Case 1.1 as the equilibrium configuration, a situation which establishes rich households in the suburb. As the population further expands, if we rule out the case of a large population shift between the communities, the growth path continues to select the Case 1 equilibrium - poor households form the city's majority and the configuration changes from Case 1.1 to Case 1.2 to Case 1.3.

At population greater than 756,662 households, if an equilibrium with the configuration of Case 1 were to exist, the boundary between rich and poor households in the suburb would be far from the metropolitan center; rich suburban households would have such high rents and commuting costs that they would gain by moving "back" to the city even at the low public service level chosen by the city residents. Poor households would be outbid in the city and would move to the suburb. The only equilibrium is Case 2.2. This incentive for rich households to move back to the city as the metropolitan area grows is qualitatively similar to the gentrification that has been observed in many large U.S. cities over the last couple of decades.

To summarize this section, our simulation suggests that, as the population grows, rich households migrate to form a new community beyond the city's jurisdictional boundary. This in turn suggests that later growth selects the equilibrium in which the poor are the majority in the city. At large populations, Case 1 cannot be supported and rich households move back into the city.

5. WELFARE COMPARISONS

5.1 Description of the Equilibria

Tables 2 and 3 present the simulation results for the equilibria in which poor and rich households form the city's majority at selected metropolitan population sizes. In each table LIne

City jurisdictional boundary: $B = 5.0$ (miles)		Equilibrium outcomes								
1. Metropolitan population, (households)	N	1,000	20,000	100,000	200,000	300,000	400,000	550,000	700,000	850,000
2. Equilibrium Case Number (label from Section 2)		city only	city only	1.1						

City jurisdictional boundary: *B = 5.0* (miles)		Equilibrium outcomes								
1. Metropolitan population, (households)	*N*	1,000	20,000	100,000	200,000	300,000	400,000	550,000	700,000	850,000
2. Equilibrium Case Number (label from Section 2)		no equil-ibrium	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.2
3. Utility of poor households ($ per year)			11,095	10,993	10,879	10,722	10,509	10,235	9,996	9,782
4. Utility of rich households ($ per year)			37,929	36,930	36,182	33,213	32,659	31,946	31,325	30,768
5. Average commuting cost ($ per year)			764	1,122	1,402	1,651	1,906	2,236	2,522	2,779
6. Average consumer surplus from public service ($/yr)			-4,583	- 4,583	- 4,583	- 4,706	- 4,918	- 5,092	- 5,192	- 5,256
7. Average rent per household ($ per year)			141	333	484	1,676	1,591	1,582	1,625	1,690
8. Boundary between income groups in city (miles)	*x*		1.38	3.09	4.37	5	5	5	5	5
9. Boundary of city development (miles)	*X*		1.38	3.09	4.37	5	5	5	5	5
10.Boundary between income groups in suburb (miles)	*y*		5	5	5	5.35	6.17	7.24	8.17	9.00
11. Boundary of suburban development (miles)	*Y*		5.14	5.66	6.25	7.05	8.14	9.55	10.77	11.87
12. Poor household's utility change if he moved from suburb to city ($ per year)			-169	- 400	- 535	- 2,422	- 2,618	- 2,871	- 3,091	- 3,289
13. Rich household's utility change if he moved from city to suburb ($ per year)			-4,100	- 3,238	- 2,646	0	0	0	0	0

Table 3: Equilibria in which the city has a majority of rich households

Reading across each table, as the metropolitan population increases, households are living further out and commuting costs increase. This in turn causes the utility achieved by poor and rich households to decrease.

The average consumer surplus from the public service is determined by the quality of the match between households and their preferred public service. For the equilibria in which poor households are the city's majority: as the metropolitan population increases from 1,000 to 400,000, the proportion of the rich households who live in the city and obtain the low public service is decreasing, and the average consumer surplus increases accordingly. At metropolitan populations above 400,000, it is poor households in the suburb who are being mismatched with the public service; as the proportion of poor households who live in the suburb increases, the average consumer surplus decreases. For the equilibria in which rich households are the city's majority: as the metropolitan population increases from 200,000, more rich households live in the suburb and obtain a low public service, and the average consumer surplus decreases accordingly.

Concerning the average rent. For the equilibria in which poor households form the city's majority, the average rent increases, reflecting the greater pressure from commuting. The increase is most dramatic as the equilibrium shifts from Case 1.2 to Case 1.3. With Cases 1.2 and 1.1, city rents are kept low, initially by the reservation rent and then by the low willingness to pay of rich households to live in the city. With Case 1.3, city rents are bid up by poor households in the suburb who are willing to pay a premium for the low public service in the city. For the equilibria in which rich households form the city's majority, there is a large increase in rent as the relevant case shifts from Case 2.1 to Case 2.2 , reflecting the increase in city rents as rich households bid to enter the city. However, at a population of 300,000 households, the average rent is so high that further increases in population cause the average rent to follow a U-curve. There is a trade-off between the increased rent paid by pre-existing households and the low rent paid by the incremental households added in the low-rent suburb. Initially the latter effect dominates and average rent falls, but at large populations the former effect dominates and average rent rises.

5.2 Normative analysis

Section 4 discusses the selection of the equilibrium of a metropolitan area. We now apply normative analysis, asking whether the selected equilibrium is the preferred equilibrium. We find that the answer depends on the population size and on the criterion used for evaluation, and that there is a trade-off between efficiency and equity. We use Table 4 in the following discussion.

City jurisdictional boundary: $B = 5.0$ (miles)

	Equilibrium outcomes								
1. Metropolitan population, (households)	1,000	20,000	100,000	200,000	300,000	400,000	550,000	700,000	850,000
SUMMARY IF POOR ARE MAJORITY IN CITY (CASE 1)									
2. Equilibrium Case Number (label from Section 2)	city only	city only	1.1	1.1	1.2	1.3	1.3	1.3	no equil-ibrium
3. Efficiency measure (Average utility plus average rent) ($ per household per year)	24,389	24,018	23,694	23,497	23,333	23,063	22,660	22,338	
4. Utility of poor households ($ per year)	12,018	11,742	11,488	11,269	10,952	9,607	9,333	9,095	
SUMMARY IF RICH ARE MAJORITY IN CITY (CASE 2)									
5. Equilibrium Case Number (label from Section 2)	no equil-ibrium	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.2
6. Efficiency measure (Average utility plus average rent) ($ per household per year)		24,653	24,294	24,014	23,643	23,175	22,672	22,286	21,965
7. Utility of poor households ($ per year)		11,095	10,993	10,879	10,722	10,509	10,235	9,996	9,782
CITY MAJORITY PREFERRED FOR:									
8. City majority for efficiency		rich	rich	rich	rich	rich	rich	poor	
9. City majority for equity		poor	poor	poor	poor	rich	rich	rich	

Table 4: Normative evaluation of equilibria as the metropolitan population increases

5.2.1 Efficiency comparisons

Our interest is in comparing the total surplus achieved in the two cases - with poor households being the city's majority and with rich households being the city's majority. Because we

2.1 all households obtain their preferred public service. Therefore, both commuting and matching favor Case 2.1, or the efficient equilibrium has rich households being the city's majority.

If the metropolitan population is 300, 000 households, the relevant comparison is between Cases 1.2 and 2.2. Case 2.2 continues to have lower commuting costs. In both configurations, some rich households receive "too little" public service. However, the number of households thus mismatched is smaller in Case 2.2.[21] Hence commuting and matching considerations continue to ensure that the efficient equilibrium is the equilibrium in which rich households are the city's majority.

If the metropolitan population is larger (400,000 households or greater in our simulations), the relevant comparison is between Case 1.3 and Case 2.2. Case 2.2 is favored for its lower commuting costs. With Case 1.3 it is some poor households who are mismatched; with Case 2.2 it is some rich households who are mismatched and, as noted earlier, matching of rich households is more important than matching of poor households. Consumer surplus is thus higher in Case 1.3. There is therefore a conflict between commuting considerations and matching considerations. At intermediate populations (400,00 and 550,000 households), the number of mismatched households is sufficiently small that commuting considerations dominate and Case 2.2 is efficient. As the metropolitan population increases, the number of mismatched households increases and matching considerations become increasingly important. If the metropolitan population is large (750,000 households), the mismatched households are sufficiently numerous that matching considerations dominate and Case 1.3 is efficient. The efficient equilibrium therefore shifts from having a majority of rich households to having a majority of poor households.

5.2.2 Equity comparisons

Our measure of equity is the Rawlsian welfare function, *max min* $[U_1, U_2]$ where U_i is the utility achieved by a household with income M_i.[22] An important difference between the efficiency and equity analyses concerns the treatment of rent. Efficiency is concerned with total surplus: any gain which accrues to landlords is included in the analysis and rent therefore is considered "as if" returned to households. Equity is concerned with the surplus accruing to poor that this gain is capitalized in the rent: the gains of matching poor households with their preferred public service go to landlords. However, in Case 2.2, these gains are not bid away to landlords. Hence equity favors Case 2.2, or the equilibrium in which rich households are the city's majority.

5.2.3 Conflict between efficiency and equity

The above discussion has shown that, at many metropolitan populations, there is a conflict between efficiency and equity as to the "goodness" of the selected equilibrium in which poor families are the city's majority. When there is no conflict between efficiency and equity, the growth path selects the equilibrium which is both inefficient and inequitable. These findings are summarized in Lines 8 and 9 of Table 4.

If the metropolitan population is small (100,000 - 300,000 households), all (or almost all) the households of one income group can live in the city and be matched with their preferred public service. Commuting considerations favor placing rich households in the city, and the equilibrium in which rich households form the city's majority is efficient. However, city rents are low and the same considerations - low commuting costs - mean that the utility of poor households is greater if poor households live in the city to form the city's majority.

If the metropolitan population is intermediate or large, (400,000 households or greater) the income group which resides in the city must also spill over into the suburb. For efficiency: initially commuting considerations dominate and favor placing rich households in the city. At very large populations, matching considerations dominate and favor placing all rich households together - in the suburb. For equity, it is important that gains to matching are not bid away to landlords, or that all poor households live in the same jurisdiction - the suburb.

6. CONCLUSION

In a monocentric urban model with two jurisdictions - a inner city and a surrounding suburb - there tend to be multiple equilibria: one in which poor households are the city's majority and one in which rich households are the city's majority. We have suggested that poor households are concentrated in the inner city because this is the equilibrium which is selected by the growth path: as the metropolitan population grows, the high willingness to pay of rich households causes them to “jump over” undeveloped land to establish a community in the suburb. For small metropolitan populations, this selected configuration is inefficient but equitable; for intermediate populations, it is inefficient and inequitable; for large metropolitan populations, it is efficient but inequitable. Although the model is necessarily stylized, we believe it highlights important trade-offs in urban policy and in the growth of metropolitan areas.

Our model has focused on the U.S. experience in which separate communities have considerable autonomy, leading to considerable variation in public service levels across communities. What supports the equilibrium in which the rich households are congregated in the suburb is their high willingness to pay to obtain their preferred public service. In contrast to the U.S., Breuckner, Thisse and Zenou. (1999) note that many European cities have higher income in the inner city than in the suburbs. We believe that an important difference between the U.S. and Europe is that, in Europe, separate communities are subject to more control from the regional government, or there is less variation in the public service level across communities. In our model, with a smaller difference in the service level between communities, rich households have a smaller incentive to “jump over” undeveloped land to form a new community in the suburb, or

APPENDIX: FORMAL

If $y = Y$, only rich households live in the suburb. If a poor household were to move to the suburb, he would achieve his higc1.0000 sT = Y

Similarly, if there are rich households in the city and in the suburb

$$0<x \text{ and } B<y: \; M_2 - b_{2s} - \qquad ; \qquad \text{(A.3b)}$$

Similarly, if there is undeveloped land in the city, the rent at the limit of development must be the reservation rent r_0. If there is no undevelope the suburb as

$$\frac{\pi(Y^2 - y^2)}{a_1} < \frac{\pi(y^2 - B^2)}{a_2}: \qquad \beta(\ldots ; \qquad \text{(A.7b)}$$

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1. Ross and Yinger (1999) survey this literature.

2. Wheaton (1976) and Sasaki (1990) provide a comparative static analysis of this equilibrium.

3. The assumption of fixed housing size greatly simplifies the problem and allows us to avoid well-known existence problems associated with stratified local public-finance equilibria (see Epple, Filimon and Romer (1984, 1993)).

4. For ease of presentation, the community is assumed to provide a public service and not a public good. The public service shows constant returns to community size. It is straightforward to change the publically-provided good from a public service to a public good.

5. Put differently, the commuting benefit to rich households of living closer to the metropolitan center outweighs the benefit they can achieve from the reduced price of land further out.

6. This assumption is consistent with the findings of Glaeser, Kahn, and Rappaport (2000).

7. A formal proof is provided in de Bartolome and Ross (2000).

8. This assumption greatly simplifies the model. We do not believe that our results depend on this assumption - what is important is that the two communities vote different public service levels.

9. Because of the specific form of the utility function, it is straightforward to change this assumption to allow each household to receive an equal share

1990, Table 1.1).

15. Using the labels described later in the text, the equilibrium at $N = 300{,}000$ in Table 3 changed from Case 1.2 to Case 1.1.

16. Although the causation is quite different, this result resembles the leapfrog development pattern that may appear in models of urban growth where some land is left vacant in the interior because its option value for future development exceeds its value in current use. For some examples in the literature, see Arnott and Lewis (1979), Capozza and Helsley (1989) and Wheaton (1982).

17. With equal numbers of poor and rich households, the rich become the majority in the city immediately the poor spill over into the suburb.

18. The "average" inner city has area 82 sq miles, or has radius 5.1 miles. This is calculated as total land area of all central places inside a metropolitan area (*Census of Population and Housing 1990*, Table 11) divided by number of metropolitan areas (*Statistical Abstract of the United States 1998,* Table 40).

19. At $N=1{,}000$ and $N=20{,}000$, the suburb is uninhabited: if a poor household moved to the suburb he would vote a low public service. The utility changes are therefore smaller than the changes of -1,392 , -1,309, ... at $N = 100{,}000$, $N= 200{,}000$, ... in which the rich are "already" in the suburb and the suburban public service is high.

20. This is the average rent paid per household. If the value quoted in the table is T ($ per household per year): because the average lot size is 0.3333 (acres per household), the average rent per acre per year is $3T$. Because the reservation rent has been set to zero, this is interpreted as the average rent premium (above the reservation rent) paid for one acre of land in the metropolitan area.

21. In Case 1.2: the number of rich households in the city is 20,273. In Case 2.2: the number of rich households in the suburb is 18,861.

22. Equity is normally interpreted as placing more weight on the welfare of the poor. For convenience of exposition, we take an extreme welfare function with all weight being placed on the welfare of the poor. If equity objective is utilitarian, $U_1 + U_2$, the basic conclusion is unchanged: there is often conflict between efficiency and equity and, when there is no conflict, growth selects the equilibrium which is inefficient and inequitable.

23. If households of income M_i do not live in the community j, we interpret b_{ij} to be the rent plus commuting cost which a household of income M_i would pay if he were to move into the community j.

24. I.e., $x < X$: $M_1 - b_{1c} - g_c \geq 0$; $0 < x$: $M_2 - b_{2c} - g_c \geq 0$; $y < Y$: $M_1 - b_1$;