DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 04-08

A Collective Household Model with Choice-Dependent Sharing Rules

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August 2004

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Abstract

This paper presents a collective household model in which there are marital gains to assortative spousal matching, individuals face a labor-leisure choice and intra-marital allocations are determined by an endogenous sharing rule that is driven by actual wage earnings. The latter two features of the model introduce the potential for inefficiently high levels of labor supply because spouses recognize that changes in their labor supply would influence not only total household income but also their respective shares in intra-household allocations. Nonetheless, when sex ratios are imbalanced or external distribution factors are not gender neutral, competition among potential spouses in the large marriage markets helps to generate maritally sustainable Pareto efficient levels of labor supply and intra-household allocations. In such cases, the sharing rule that supports the maritally sustainable and Pareto efficient equilibrium outcome is also unique for each couple along the assortative order.

Keywords: The Collective Model, Marriage, Matching, Household Labor Supply.
JEL Classification Numbers: C78, D61, D70.

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1. Introduction

The traditional approach to analyze household choices takes the family as the relevant decision-making unit.[1] The collective household model provides an alternative to this approach by treating the individual members of the family{not the family as a whole{as the core decision-makers.[2] Starting in the early 1990s, the empirical literature began to provide strong support for the notion that relative spousal incomes matter for family decisions and intra-household allocations.[3] Consequently, the collective approach to household decision-making has emerged as the compelling theoretical tool for analyzing the economics of the family.

The collective model is based on the premise that external distribution factors such as the sex ratios in the markets for marriage and the distributions of income within the households determine the intra-marital sharing rules. It requires that the latter do not depend on variables that enter spousal choice sets. But what if sharing rules, to some extent, do depend on spousal choices made during the marriage? Then, there are two seemingly fundamental obstacles. First, it is not clear how one would model, for example, the household labor supply choices in a framework in which individuals value leisure and the marital decision-making power of the spouses depends on their relative actual labor incomes. In that case and in the absence of binding commitments prior to the formation of marriage, the spousal levels of labor supply and leisure could be determined via a decision-making process that is non-cooperative and competitive in nature. Such a solution method could make it less likely that there is specialization within the household. Then would modeling the household labor supply as the outcome of a non-cooperative process be reasonable and empirically consistent?

Second, a vital building block of the collective model is Pareto e ciency. As demonstrated by Chiappori (1988, 1992), Pareto optimality enables one to recover the underlying preference structure of the individuals within the household as well as the implicit sharing rule that in uences the intra-household allocations among di erent family members.[4] For existing households, e ciency is a robust assumption as long as the sharing rules consistent with the collective model are primarily driven by external factors, such as the sex ratios in the markets for marriage, divorce legislation, and potential (not actual) spousal incomes. Pareto e ciency could become suspect, however, in models where the marital decision-making power of spouses depends on their actual labor incomes relative to that of their partners. Then, it is quite possible that the household labor supply would be ine ciently high as spouses would recognize that their labor supply choices in uence not only total household income but also their decision-making power within the marriage.

The conventional models of the collective household typically avoid these complications by either ruling out leisure from individual preferences or assuming that the incomes relevant for intra-marital allocations are those that the spouses could earn entering a marriage{not those that the husband and the wife actually do earn once all labor supply, household production and leisure choices are made.[5] For instance, if two stay-home wives have di erent levels of education, they either value leisure and are compensated di erently in their marriages ceteris paribus, or have no preference for leisure and are compensated roughly similarly.

Since some empirical studies that nd support for the collective model focus on the observed levels of total household earnings and how those are distributed within the household, they suggest that actual spousal earnings do matter for intra-marital allocations.[6] Hence, it is important to address whether sharing rules that depend on

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4 *conditional efficiency.* *conditional*

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spousal choices and the possibility for strategic spousal behavior during marriage alters{ or even worse invalidates{the collective household approach.

In what follows, I present a collective household model in which there are marital choices by relying on an intra-household sharing rule. Its special case the marital bargaining model generates the same feature via spousal Nash-bargaining weights. Among the earliest examples of the collective models are Becker (1981), Chiappori (1988, 1992), labor supply choices influence intra-marital allocations.

This paper is most similar to Becker-Murphy (2000), Browning-Chiappori-Weiss (2003) and Iyigun-Walsh (2004). All three represent the early attempts to broaden the collective approach to cover aspects of household formation that precede marriage.[7] Becker-Murphy and Browning-Chiappori-Weiss share similarities in that they both merge the collective household model with marital sorting to explore the implications of spousal matching. In both contributions, however, the endowment each spouse brings to the marriage is taken as given. Iyigun and Walsh extend the collective model to cover pre-marital investments and marital sorting. They find that matching in the marriage markets helps to generate unique sharing rules that support unconditionally Pareto efficient outcomes (where both intra-household allocations and pre-marital choices are Pareto efficient).

Preferences are de ned over the consumption of a single good and leisure, c_i and $y_i \quad l_i$ respectively, where l_i denotes individual i's endogenously-determined labor supply. For males and females, preferences are represented by the following inter-temporal utility functions respectively:

$$U = u(y_m \quad l_m) + u(c_m) ; \tag{1}$$

and

$$V = v(y_f \quad l_f) + v(c_f) ; \tag{2}$$

where the functions U and V satisfy $u^0; v^0 > 0; u^{00}; v^{00} < 0$, and the other neo-classical Inada restrictions.

The marital production technology is given by $h(l_m; l_f)$. If a man with a labor supply of l_m remains single, his intra-temporal output is given by $h(l_m; 0)$ and if a woman with an income of l_f remains single, her intra-temporal output is given by $h(0; l_f)$. I assume that the function $h(l_m; l_f)$ is increasing in l_m and l_f and that $h(0; 0) = 0$.

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$$u[h(l_m^s; 0)] \qquad \text{and} \qquad v[h(0; l_f^s)]. \qquad (3)$$

For those individuals who remain single, the optimal levels of labor supply, l_i^s, $i = f; m$, are

$$l_i^s = \begin{cases} \arg\max \; U = u(y_m \quad l_m^s) + u[h(l_m^s; 0)] & \text{if } i = m, \\ \arg\max \; V = v(y_f \quad l_f^s) + v[h(0; l_f^s)] & \text{if } i = f. \end{cases} \qquad (4)$$

The optimal labor supply of single men and women respectively satisfy the following rst-order conditions:

$$u^0(y_m \quad l_m^s) = u^0$$

where g represents the common gain from marriage that is unrelated to spousal incomes.[10] Note that equation (6) holds only for couples that match with each other in the marriage market (and not for those who have chosen not to match with each other).[11] Due to the super modularity of the marital output function, also keep in mind that, 8 $(l_m, l_f) >> 0$, $h(0; l_f) + h(l_m; 0) < h(l_m; l_f)$. Therefore, the function $h(l_m, l_f)$ explicitly incorporates the \gains" from marriage.

The couple (y_m, y_f) plays a non-cooperative Nash game in which each spouse takes as given the other's actions. Let the labor supply response function of a husband be de ned as:

$$\begin{aligned} l_m(l_f) &= \arg\max \; U(l_m \,|\, l_f) \\ &= \arg\max \; fu(y_m \quad l_m) + u[c_m(h(l_{m;}\; l_f))]g \,. \end{aligned} \tag{7}$$

In similar fashion, let the labor supply of a wife as a function of that of her husband be de ned as:

$$\begin{aligned} l_f(l_m) &= \arg\max \; V(l_f \,|\, l_m) \\ &= \arg\max \; fv(y_f \quad l_f) + v[c_f(h(l_{m;}\; l_f))]g \,. \end{aligned} \tag{8}$$

The related rst-order conditions are

$$u^0(c_m)c^0_m = u^0(y_m \quad l_m) \tag{9}$$

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[10] g g c_m c_f U V:

[11] $c_m \; l^*_m \quad c_f \; l^*_f \; > h \; l^*_m; l^*_f$ g

2. $8\ y_m, y_f\ 2\ [0; Y]; l_m = \ (l_f)$ and $l_f = \ (l_m)$;

3. $8\ y_m\ 2\ [0; Y]; y_f = \arg\max f u(y_m \quad l_m) + u[h(l_m; l_f) + g \quad c_f(l_f)] g$;

4. $8\ y_f\ 2\ [0; Y]; y_m = \arg\max f v(y_f \quad l_f) + v[h(l_m; l_f) + g \quad c_m(l_m)] g$.

Part 1 of the de nition is the marriage market-clearing condition which guarantees that, by assortative matching, each husband that is endowed with y_m or more will be able to match with a spouse who is endowed with at least y_f. It generates the following spousal matching functions:

$$y_m = \quad f1 \quad r(1 \quad H(y_f)]g \qquad (y_f) \tag{11}$$

and,

$$y_f = \left\{1 \quad \frac{1}{r}(1 \quad G(y_m)]\right\} \qquad (y_m) \tag{12}$$

where G^{1} and H^{1}. Note that either of the functions (y_f) and (y_m) fully describe the nature of spousal matching.

Part 2 of the de nition re ects the fact that, once married, couples play a v9fbh-]TJ-29.888-19.9

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$$v'(y_f - l_f)\frac{\partial l_f}{\partial l_m}\frac{\partial l_m}{\partial y_m} = v'[h(l_m; l_f) + g - c_m(l_m)]\left\{h_2\frac{\partial l_f}{\partial l_m}\frac{\partial l_m}{\partial y_m} + [h_1 - c'_m(l_m)]\frac{\partial l_m}{\partial y_m}\right\} . \tag{14}$$

Equation (13) implies that there are both direct and indirect effects of a husband with an endowment of y_m marrying a wife with y_f. The direct effect is captured by the last term on the right hand side of (13) and it represents the impact of the best-response labor supply of the wife on the marital gain of her husband. If the wife receives less (more) than her marginal contribution to the marriage, then the direct effect of a marginal increase in her labor supply on her husband is positive (negative). There are two indirect effects of a husband with y_m marrying the wife with y_f. The best-response labor supply of this husband influences his leisure, captured by the term on the left hand side of (13), as well as his marital gain, denoted by the first term on the right hand side of equation (13). The interpretation of equation (14) is, of course, similar to that of (13).

Note that, $\forall\,(y_m, y_f)$; the rational expectations equilibrium implicitly defines two distributions functions $\hat{G}(l_m)$ and $\hat{H}(l_f)$ such that $1 - \hat{G}(l_m) = r[1 - \hat{H}(l_f)]$. On that basis and consistent with the notation above, we can re-define the spousal matching functions as $l_m = \hat{\ }(l_f)$ and $l_f = \hat{\ }(l_m)$. In Figure 2, I rely on these labor supply distributions and depict two possible rational expectations equilibria that could emerge in the marriage market.[16] The labor supply of the men are drawn on the horizontal axis and those of the women are on the vertical axis. The two upward-sloping dashed lines represent two different equilibrium matching functions $\hat{\ }(l_m)$ for a given rule of intra-marital sharing of spousal consumption. The upward convex curves are the indifference curves of the husbands and those that are convex downward are the indifference curves of the wives. Both types of indifference curves incorporate the sharing rules associated with each potential spousal match. Due to the assortative matching equilibrium, couples for which the wife has a higher initial endowment, y_f, work more than those for which

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the wife has a lower initial endowment. If distributional factors favor women more than they do men then, for a given sharing arrangement of spousal consumption within the households, the equilibrium matching function will tend to shift to the right leading to a higher labor supply by the husbands and less by the wives. For each matched couple, the tangency point of the indi erence curves of husbands and wives also correspond to the intersection point of the labor supply response functions $l_m = \quad ^{1}(l_f)$ and $l_f = \quad (l_m)$ (which were originally depicted in Figure 1). One such point is identi ed as the point A in Figure 2.

[Figure 2 about here.]

6. The Pareto E cient Frontier

For the couple (y_m, y_f), the unconditionally e cient levels of labor supply and intra-household allocations of consumption can be determined by solving the following maximization problem:

$$\max_{f l f;\, l \;\; ;c \;\; ;c \;\; g} U = u(y_m \quad l_m) + u(c_m) \tag{15}$$

subject to:

$$V = v(y_f \quad l_f) + v(c_f) \qquad V , \tag{16}$$

$$c_m + c_f \qquad h(l_m;\, l_f) + g \tag{17}$$

and,

$$l_m \qquad y_m \quad \text{and} \quad l_f \qquad y_f : \tag{18}$$

The four rst-order conditions to this problem yield

$$u'(y_m - l_m) = u'(c_m)\, h_1(l_m, l_f) \; ; \tag{19}$$

and,

$$v'(y_f - l_f) = v'(c_f)\, h_2(l_m, l_f) \; : \tag{20}$$

Utilizing the restrictions imposed on this problem, these conditions can be re-written as

$$\frac{u'(y_m - l_m)}{u'[c_m(l_m)]h_1[l_m, \;\; (l_m)]} = \frac{v'(y_f - l_f)}{v'[c_f(l_f)]h_2[\;\; (l_f), l_f]} \quad . \tag{21}$$

Along the Pareto efficient frontier, equation (21) equates spouses' ratios of marginal utility of leisure to marginal utility of consumption. When combined with the endowment constraint, equation (17), the first order conditions of equations (19) and (20) determine the Pareto efficient frontier. Along this frontier, the wife's utility constraint, equation (16), ties down the allocation associated with the wife attaining utility equal to V.

7. Equilibrium Sharing Rules and Marital Stability

We are now in position to address whether the marital matching process and the subsequent allocations of intra-marital consumption and leisure satisfy Pareto efficiency. The sharing rules that hold in equilibrium and that are therefore maritally sustainable need to be compatible with equations (9), (10), (13) and (14), all of which need to be satisfied for all married couples along the assortative order.

Combining these four equations and rearranging a bit, we get

$$\frac{u'(y_m - l_m)}{u'[c_m(l_m)]c_m'(l_m)} = \frac{v'(y_f - l_f)}{v'[c_f(l_f)]c_f'(l_f)} = 1 \,, \tag{22}$$

$$\frac{u'(y_m - l_m)}{u'[c_m(l_m)]} = \frac{1}{\frac{\partial l}{\partial l}\frac{\partial l}{\partial y}} \left\{ h_1(l_m, l_f)\frac{\partial l_m}{\partial l_f}\frac{\partial l_f}{\partial y_f} + \left[h_2(l_m, l_f) - c_f'(l_f)\right]\frac{\partial l_f}{\partial y_f} \right\} \tag{23}$$

and,

$$\frac{v^0(y_f \qquad I_f)}{v^0[c_f(I_f)]} = \frac{1}{\frac{\partial I}{\partial I} \quad \partial I} \qquad f$$

[Figure 3 about here.]

When the sex ratio, r, is equal to unity and the external distribution factors are neutral, all individuals marry and every husband and wife with a strictly positive endowment exceeds his or her reservation utility level.[18] Then, we cannot move beyond equation (25) and all we can conclude is that there exists a continuum of maritally sustainable intra-household sharing rules{only one of which is Pareto e cient.[19] [20]

In contrast, consider a case in which $r > 1$ or external distributions heavily favor men so that, among couples in the lowest assortative rank (when $r > 1$, those with $y_f^0 > y_m^0 = 0$), wives receive their reservation utility, which equals $v(y_f \quad l_f^s) + v[h(0; l_f^s)]$. In that case, we establish that equation (5) holds for married women in the lowest assortative rank. That is $v^0(y_f^0 \quad l_f^s) = v^0[h(0; l_f^s$

this couple, let $h_1(\theta_m, \theta_f) \neq c'_m(\theta_m)$ and $h_2(\theta_m, \theta_f) \neq c'_f(\theta_f)$. Now consider the analog of equation (14) for the wife with the endowment of $\bar{y}_f$. If she marries a husband with $\bar{y}_m$ and gets a share in marriage associated with $h_2(\theta_m, \theta_f) \neq c'_f(\theta_f)$ and $h_1(\theta_m, \theta_f) \neq c'_m(\theta_m)$, we have

$$\frac{v'(\bar{y}_f \quad \theta_f)}{v'[c_f(\theta_f)]} = \frac{1}{\frac{\partial\theta}{\partial\theta}\,\frac{\partial\theta}{\partial\bar{y}}} \Big\{ h_{2\sim}$$

$\tilde{y}_f$) is maritally sustainable if the intra-marital allocations for that pair are consistent with $h_1(\tilde{y}_m, \tilde{y}_f) \geq c^0_m(\tilde{y}_m)$ and $h_2(\tilde{y}_m, \tilde{y}_f) \geq c^0_f(\tilde{y}_f)$. Put di erently, such a pairing would be maritally sustainable if and only if it yields Pareto e cient intra-marital allocations (i.e. the conditions $h_1(\tilde{y}_m, \tilde{y}_f) = c^0_m(\tilde{y}_m$

[Figure 4 about here.]

What if the labor supply functions described by equations (9) and (10) generate multiple labor supply equilibria for each couple? When either wives or husbands in the lowest assortative order receive their reservation levels of utility (as would be the case when $r \neq 1$), it is clear that the above reasoning (which ensures that, 8 $(y_m; y_f)$; $h_1(l_m, l_f) = c^0_m(l_m)$ and $h_2(l_m, l_f) = c^0_f(l_f$

di erently, the marital matching functions $\hat{}(I_f)$ and $\hat{}(I_m)$ are such that, 8 (I_m, I_f), I_m = $(I_f) = \hat{}(I_f)$ and I_f = $(I_m) = \hat{}(I_m)$.

8. An Example

For simplicity, let the marital gain, g, equal zero and the marital production function be given by

$$h(I_m; I_f) = I_m + I_f + I_m I_f . \qquad (31)$$

Suppose that the preferences of males and females are represented by the following inter-temporal utility functions respectively:

$$U = \quad \ln(y_m \quad I_m) + (1 \quad) \ln(c_m) ; \qquad (32)$$

and

$$V = \quad \ln(y_f \quad I_f) + (1 \quad) \ln(c_f) ; \qquad (33)$$

where , 2 (0; 1) and the consumption levels of men and women are given by

$$c_m + c_f \quad I_m + I_f + I_m I_f . \qquad (34)$$

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We can now explore the outcomes under three di erent cases:

1. If r = 1 so that the measures of men and women in the marriage market are identical, all individuals marry. As a result, we can establish that, 8 (y_m, y_f), y_m = y_f. The analogs of equations (9) and (10) correspond to the following:

$$\frac{c_m(l_m)}{y_m \quad l_m} = (1 \qquad) \, c^0_m(l_m) \; , \tag{35}$$

$$\frac{c_f(l_f)}{y_f \quad l_f} = (1 \qquad) \, c^0_f(l_f) : \tag{36}$$

And the analog of (25) is

$$1 + l_f \qquad c^0_m(l_m) = \qquad [1 + l_m \qquad c^0_f(l_f)$$

and,

$$c_f(l_f) = \int_0^l (1 + t)\, dt = l_f + (l$$

$$I_f = I_f^s$$

between marrying him and remaining single. Hence, for " ! 0^+) $\check{t}_f$(") ! 0^+, this new spousal match would be dominating for the husband with the endowment of $\check{y}_m$, in contradiction of the fact that the existing marriage market equilibrium is stable. Only if the intra-marital sharing rule yields the Pareto e cient outcomes so that, 8 ($\check{y}_m$; $\check{y}_f$), $h_1(\check{t}_m; \check{t}_f) = 1 + \check{t}_f = c^0_m$ and $h_2(\check{t}_m; \check{t}_f) = 1 + \check{t}_m = c^0_f$, would the existing assortative marriage market equilibrium be stable. Moreover, given the continuity of the endowment distributions over the support [0; Y], the process just described would yield the unique sharing rule that supports the Pareto e cient intra-marital allocations and levels of spousal labor supply for all marriages along the assortative order. Then, using equations (11), (12), (29) and (30), we can derive that, for r < 1,

$$c_m(l_m) = \frac{1}{r}\int_0^{l} (2r \quad 1 + s)\, ds = 2l_m \quad \frac{l_m}{r} + (l_m \quad {}_m) = \int ds \quad ds$$

$$c_m(l_m) \quad = \quad \frac{1}{r}\int_{l^0}^{l} (2r \qquad 1 \; + \; s)\, ds \qquad (45)$$

$$= \quad 2l_m \qquad \frac{l_m}{r} \; + \; \frac{(l_m)^2}{2r} \; + \; \frac{3r}{2} \; + \; \frac{1}{2r} \qquad 2\,,$$

and,

$$c_f(l_f) \; = \; \int_0^{l} (2 \qquad r \; + \; rt)\, dt \; = \; (2 \qquad r)l_f \; + \; \frac{r}{2}(l_f)^2\,. \qquad (46)$$

Again, the optimal spousal levels of labor supply could be derived as in case 1.

9. Conclusion

In analyzing intra-marital family decisions, the collective household model treats each individual family member{as opposed to the whole family{as the relevant decision making unit. Empirical studies carried out in the last decade or so have provided consistent support for the idea that relative spousal incomes matter for family decisions and intra-household allocations. Hence, the collective approach to household decision-making has emerged as the compelling theoretical tool for analyzing the economics of the family.

The collective model relies on the assumption that external distribution factors such as the sex ratios in the markets for marriage and the distributions of income within the households determine the intra-marital sharing rules. Conventionally, it requires that the intra-marital sharing rules do not depend on internal distribution factors; that is, variables that enter spousal choice sets. As a consequence, either leisure is ruled out from individual preferences or the incomes relevant for intra-marital allocations are assumed to be those that the spouses could earn entering a marriage (and not those that the husband and the wife actually do earn once all labor supply, household production and leisure choices are made). But what if sharing rules depend on choices individuals make during the marriage? To take an example, how should we treat cases in which leisure enters individual preferences and intra-marital sharing rules are in uenced by the household distribution of actual wage earnings? Then, there are at least two important in the marriage markets are not equal to unity or external distribution factors (such as marriage and divorce legislation) are not gender neutral, marriage market competition among potential spouses helps to generate maritally sustainable and Pareto efficient levels of labor supply and spousal consumption. In such cases, the sharing rule that supports the efficient, maritally sustainable equilibrium is also unique for each couple along the assortative order.

In sum, I have identified that neither strategic spousal interactions nor the endogeneity of intra-marital sharing rules with respect to spousal choices made during the marriage need to be accounted for if the marriage markets are large and the external distribution factors are asymmetric. Then, the efficiency of household choices are generally restored because marriage market competition helps to ensure that each spouse is compensated according to his or her marginal contribution to the marriage.

Manser, M. and M. Brown. (1980). \Marriage and Housheold Decision-Making: A Bargaining Analysis," International Economic Review, 21, February, 31-44.

McElroy, M. B. and M. J. Horney. (1981). \Nash-Bargained Decisions: Towards a Generalization of the Theory of Demand," International Economic Review, 22, June, 333-49.

Peters, M. and A. Siow. (2002). \Competing Pre-Marital Investments," Journal of Political Economy, 110 (3), 592-608.

Samuelson, P. (1956). \Social Indi erence Curves," Quarterly Journal of Economics, 70 (1), 1-22.

Sen, A. (1983). \Economics and the Family," Asian Development Review, 1, 14-26.

Udry, C. (1996). \Gender, the Theory of Production, and the Agricultural Household," Journal of Political Economy, 104 (5), October, 1010-46.

Figure 3: The Marital Contract Curve and the Efficient Frontier

l_f

$\psi(l_m)$

y_f

Figure 4: The Marital Contract Curve and the Efficient Frontier