DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 05-02

Closing International Real Business Cycle Models with Restricted Financial Markets

Martin Boileau
Department of Economics and CIRPÉE, University of Colorado at Boulder
Boulder, Colorado

Michel Normandin
Department of Economics and CIRPÉE, HEC Montréal
Montréal, Que, Canada

March 2005

Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

Closing International Real Business Cycle Models with Restricted Financial Markets

Martin Boileau* and Michel Normandin

March 2005

Abstract

Several authors argue that international real business cycle (IRBC) models with incomplete financial markets offer a good explanation of the ranking of cross-country correlations. Unfortunately, this conclusion is suspect, because it is commonly based on an analysis of the near steady state dynamics using a linearized system of equations. The baseline IRBC model with incomplete financial markets does not possess a unique deterministic steady state and, as a result, its linear system of difference equations is not stationary. We show that the explanation of the ranking of cross-country correlations is robust to modifications that ensure a unique steady state and a stationary system of linear difference equations. We find, however, that the modifications affect the quantitative predictions regarding key macroeconomic variables.

JEL Classification Codes: F32, G15.
Keywords: Incomplete markets, stationarity, cross-country correlations, wealth effects.

* Corresponding author. Boileau: Department of Economics and CIRPÉE, University of Colorado, 256 UCB, Boulder CO 80309 USA. Tel.: 303-492-2108. Fax: 303-492-8960. E-mail: martin.boileau@colorado.edu. Normandin: Department of Economics and CIRPÉE, HEC Montréal, 3000 Chemin de la Côte-Sainte-Catherine, Montréal Que. H3T 2A7 Canada. Tel.: 514-340-6841. Fax: 514-340-6469. E-mail: michel.normandin@hec.ca. Normandin acknowledges financial support from FQRSC and SSHRC.

1. Introduction

The international real business cycle (IRBC) model with incomplete international financial markets is successful at reconciling predicted business cycle moments with empirical moments. In particular, the IRBC model with trade in a one-period bond driven by shocks that are highly persistent and that do not spill over international boundaries solves the *quantity anomaly*. This anomaly, coined by Backus, Kehoe, and Kydland (1995), refers to the inability of the IRBC model with complete markets to correctly predict that the cross-country correlation of output is larger than the cross-country correlation of consumption. Baxter and Crucini (1995) argue that the IRBC model with incomplete markets solves the quantity anomaly because of an important differential wealth effect. In the complete markets model, a rise in home productivity generates a small increase in wealth at home and a large increase in wealth abroad. This arises because complete international financial markets ensure perfect risk sharing. The result is that home and foreign consumption fluctuations are highly correlated. In the incomplete markets model, however, the rise in home productivity generates a large increase in wealth at home, but only a small increase in wealth abroad. This arises because financial markets do not ensure perfect risk sharing. The result is that home and foreign consumption fluctuations need not be highly correlated.

Unfortunately, these conclusions are suspect because they are generated from an analysis of the model's near steady state dynamics. That is, most studies use a linear approximation method similar to that of King, Plosser, and Rebelo (2002). The method requires that the system of equations that characterizes the equilibrium be linearized around the deterministic steady state, and that the resulting system of linear difference equations be solved. The problem is that the deterministic steady state of the baseline IRBC model with trade in a one-period bond is not unique. As a consequence, the resulting system of linear difference equations is not stationary. At first glance, the non-stationarity is a serious flaw: it undermines the study of near steady state dynamics.

Our objective is to verify whether the ability to solve the quantity anomaly is robust to specifications of the model that resolve the non-uniqueness of the deterministic state and the resulting non-stationarity of the system of linear difference equations. The stationary models add a stationarity inducing modification of the baseline non-stationary model. Although not our objective, it is also possible to verify the robustness by using alternative approximation methods, as in Kehoe and Perri (2002) and Kim, Kim, and Levine (2003). Note as well that the flaw applies not only to IRBC models, but to all dynamic, stochastic, multi-agents general equilibrium models with incomplete financial markets.

Our analysis is related to that of Kim an Kose (2003) and Schmitt-Grohé and Uribe (2003). They study the dynamics of non-stationary and stationary small open economy real business cycle models. Our analysis, however, focuses on two-country IRBC models and leads to a different conclusion. They show that the different stationarity inducing modifications do not affect the quantitative predictions regarding the behavior of key macroeconomic variables. Thus, they conclude that researchers should select the modification based solely on computational convenience. In contrast, we find that the different modifications have important effects on the quantitative predictions.

We proceed as follows. In Section 2, we present the baseline two-country IRBC model with trade in a one-period bond and its calibration. We show that the model's deterministic steady state is not unique and that the linearization method yields a non-stationary system of linear difference equations. For completeness, we also show that the non-uniqueness and non-stationarity do not occur in the complete markets IRBC model.

In Section 3, we present 5 incomplete markets IRBC models that generate a unique steady state and a stationary system of linear difference equations. The first model assumes that the consumer's subjective discount factor is endogenous. The second model also assumes that the consumer's subjective discount factor is endogenous. In this case, however, the consumer does not internalize the effects of his choices on the discount factor. The third model assumes a debt elastic supply of international assets. The fourth model assumes that consumers face quadratic portfolio costs. Finally, the fifth model assumes that consumers directly care about their asset holdings.

In Section 4, we present our numerical results. First, we document that baseline and stationary incomplete markets models driven by shocks that are highly persistent and that do not spill over international boundaries solve the quantity anomaly. The models driven by shocks that spill over international boundaries, however, do not solve the quantity anomaly. Second, we find that the business cycle moments and impulse responses generated by the different models differ only when shocks are persistent and do not spill over. Thus, the quantitative predictions differ only when the models solve the quantity anomaly. Third, we find that the debt elastic interest rate model and the quadratic portfolio costs model outperform the other stationary models in the sense that they generate business cycle moments that match the empirical moments more closely. Fourth, we find that baseline and stationary models generate a similar wealth effect, but dissimilar price (wages and interest rate) effects. Finally, we show that the ability to solve the quantity anomaly relies on the ability to change the supply of physical capital, but not much on the ability to change the supply of labor. This occurs because of the price effects, and especially of the interest rate effect.

2. A Statement of the Problem

To illustrate the problem, we construct a two-country, dynamic, general equilibrium model with trade in a homogenous good and in a one-period bond. The model is similar to those in Baxter and Crucini (1995) and Kollmann (1996). In what follows, we only describe the home economy, but the foreign economy is symmetric up to country specific productivity shocks. Foreign country variables are identified by an asterisk.

2.1 The Baseline Incomplete Markets (IM) Model

In the IM model, the home economy is populated by a representative consumer and a representative firm. The consumer's expected lifetime utility is

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t,n_t)\right], \tag{1}$$

where E_t is the conditional expectation operator, c_t is consumption, n_t is employment,

$$u(c_t,n_t)=\left[c_t^{\eta}(1-n_t)^{1-\eta}\right]^{\gamma}/\gamma, \tag{2}$$

$0 < \beta < 1$, $\eta > 0$, and $\gamma \leq 1$.

The consumer's budget constraint is

$$c_t + x_t + q_t^w b_{t+1} = w_t n_t + r_t^k k_t + b_t, \tag{3}$$

where x_t denotes investment, w_t is the wage rate, r_t^k is the rental rate of capital, k_t is the capital stock, b_t is the stock of one-period bond, and q_t^w is the world price of the one-period bond. The capital stock evolves according to

$$k_{t+1} = \phi(x_t/k_t)k_t + (1-\delta)k_t \tag{4}$$

where

$$\phi(x_t/k_t) = \frac{\omega_1}{1-1/\xi}\left(\frac{x_t}{k_t}\right)^{1-1/\xi} + \omega_2, \tag{5}$$

$0 < \delta < 1$ and $\xi > 0$. Also, ω_1 and ω_2 are set so that $\phi(x/k) = \delta$ and $\phi_\delta(x/k) = 1$ in the deterministic steady state, where ϕ_{delta} is the derivative of the function $\phi(\cdot)$ with respect to x/k. The function $\phi(\cdot)$ implies an adjustment cost, and ξ is the elasticity of investment with respect to Tobin's q.

The firm's profits are

$$y_t - w_t n_t - r_t^k k_t, \tag{6}$$

where y_t denotes the firm's output. As is standard, output is produced with the constant return to scale technology

$$y_t = z_t k_t^\alpha n_t^{1-\alpha}, \tag{7}$$

where z_t is the level of total factor productivity and $0 < \alpha < 1$.

The model is closed by the asset market clearing condition

$$b_t + b_t^* = 0. \tag{8}$$

Finally, the stationary stochastic process that drives the level of productivity is

$$\begin{pmatrix} \ln(z_t) \\ \ln(z_t^*) \end{pmatrix} = \begin{pmatrix} \rho & \nu \\ \nu & \rho \end{pmatrix} \begin{pmatrix} \ln(z_{t-1}) \\ \ln(z_{t-1}^*) \end{pmatrix} + \begin{pmatrix} \epsilon_t \\ \epsilon_t^* \end{pmatrix}, \tag{9}$$

where ρ measures the persistence of productivity shocks and ν measures the degree of international spillovers. The vector $\mathbf{e}_t = (\epsilon_t \quad \epsilon_t^*)'$ contains innovations with covariance matrix

$$\Sigma = \begin{pmatrix} \sigma^2 & \psi \\ \psi & \sigma^2 \end{pmatrix}.$$

The competitive consumer chooses consumption, employment, capital and bond holdings to maximize expected lifetime utility (1) subject to the constraints (3) and (4). The competitive firm hires labor and capital to maximize profits (6) subject to the production technology (7). The set of first-order conditions of the consumer's and firm's problems, as well as the asset market clearing condition form the system of equations that characterizes the symmetric equilibrium. The system includes home and foreign variants of

$$\lambda_t = u_{ct}, \tag{10.1}$$

$$u_{nt} = -\lambda_t(1-\alpha)y_t/n_t, \tag{10.2}$$

$$q_t^w = \beta E_t\left[\lambda_{t+1}\right]/\lambda_t, \tag{10.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta E_t\left[\lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}}\left(\phi_{t+1} - \phi_{\delta t+1}\frac{x_{t+1}}{k_{t+1}} + 1 - \delta\right)\right], \tag{10.4}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{10.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta)k_t, \tag{10.6}$$

$$y_t = c_t + x_t + q_t^w b_{t+1} - b_t, \tag{10.7}$$

as well as

$$b_t + b_t^* = 0. \tag{10.8}$$

Here, u_{ct} and u_{nt} are the partial derivatives of $u(c_t, n_t)$ with respect to c_t and n_t, and λ_t is the multiplier associated with the budget constraint (3). Equation (10.1) relates the multiplier λ_t to the marginal utility of consumption. Equation (10.2) equates the marginal rate of substitution between employment and consumption to the marginal product of labor. Equation (10.3) equates the marginal cost of purchasing a unit of the one-period bond to its discounted marginal benefit. Equation (10.4) equates the marginal cost of investment in physical capital to its discounted marginal benefit. Equation (10.5) is the production technology. Equation (10.6) is the capital accumulation. Equation (10.7) is the national income identity. Finally, equation (10.8) is the asset market clearing condition.

The system (10) has 15 independent equations. These equations must solve for 7 home variables (y_t, c_t, n_t, x_t, k_t, b_t, and λ_t), 7 foreign variables (y_t^*, c_t^*, n_t^*, x_t^*, k_t^*, b_t^* and λ_t^*), and 1 asset price (q_t^w).

2.2 The Problem

As is standard, the equilibrium system (10) does not possess an analytical solution. Most IRBC studies go on to provide an approximate solution using the method described in King, Plosser, and Rebelo (2002). This method approximates the dynamics of the economy near its deterministic steady state. To do so, the equations that characterize the equilibrium are linearized around the deterministic steady state, and the resulting linear difference equations system is solved as in Blanchard and Kahn (1980).

Unfortunately, the deterministic steady state of the baseline model is not unique. That is, the steady state is characterized by home and foreign variants of

$$\lambda = u_c, \tag{11.1}$$

$$u_n = -\lambda(1-\alpha)y/n, \tag{11.2}$$

$$q^w = \beta, \tag{11.3}$$

$$1 = \beta\Big[\alpha\frac{y}{k} + 1 - \delta\Big], \tag{11.4}$$

$$y = zk^\alpha n^{1-\alpha}, \tag{11.5}$$

$$x = \delta k, \tag{11.6}$$

$$y = c + x + (q^w - 1)b, \tag{11.7}$$

as well as

$$b + b^* = 0. \tag{11.8}$$

Clearly, the deterministic steady state system (11) only has 14 independent equations, but must solve 15 variables. This occurs because the deterministic version of the bond pricing equation (10.3) of the home and foreign consumers collapse to an identical equation (11.3) in the deterministic steady state. More precisely, the home and foreign bond pricing equations are $q_t^w = \beta E_t[\lambda_{t+1}]/\lambda_t$ and $q_t^w = \beta E_t[\lambda_{t+1}^*]/\lambda_t^*$. These equations both collapse to $q^w = \beta$ in the deterministic steady state.

Admittedly, it is possible to choose a particular steady state amongst the set of possible solutions to the system (11). For example, it is common practice to assume that the symmetric deterministic steady state involves $b = b^* = 0$. Unfortunately, this yields another problem. Namely, the linear dynamic system that describes the behavior of the model's predetermined state variables is not stationary.

To clarify the non-stationarity problem, we apply the numerical linearization method. To do so, we first assign values to all parameters. We follow Backus, Kehoe, and Kydland (1992) and set $\beta = 0.99$, $\gamma = -1$, $\delta = 0.025$, and $\alpha = 0.36$. We set η to ensure that steady state hours worked are $n = 0.3$. We also set ξ to ensure that the ratio of the standard deviations of detrended investment to the standard deviations of detrended output is realistic, where the trend is removed using the Hodrick-Prescott filter. The realistic relative volatility of investment is 3.27.

In addition, we use two different parametrizations for the shock process. We do so because Baxter and Crucini (1995) argue that the IM model is very sensitive to the parameters that controls persistence (ρ) and international spillovers (ν). The first parametrization corresponds to the process in Backus, Kehoe, and Kydland (1992). The BKK shock process assumes a small value of ρ and a large value of ν: $\rho = 0.906$ and $\nu = 0.088$. The second parametrization is in the spirit of Baxter and Crucini (1995). The BC shock process assumes a large value of ρ and a small value of ν: $\rho = 0.999$ and $\nu = 0$. The remaining parameters take the values found in Backus, Kehoe, and Kydland (1992): $\sigma = 0.00852$ and $\psi = 0.258\sigma^2$.

We also simplify the system of equations (10) as in Baxter and Crucini (1995). First, we use the home version of equation (10.3) and equation (10.8) to substitute out q_t^w and b_t^*. Second, we use our solution for q_t^w to rewrite the foreign version of (10.3) as $E_t\left[\lambda_{t+1}\right]/\lambda_t = E_t\left[\lambda_{t+1}^*\right]/\lambda_t^*$. Third, we sum the home and foreign versions of (10.7) to obtain the goods market clearing condition $c_t + c_t^* + x_t + x_t^* = y_t + y_t^*$, and keep only the home version of (10.7). Finally, we use the home and foreign versions of equation (10.1) to substitute out λ_t and λ_t^*.

Finally, the dynamic system is linearized around the selected deterministic steady state. The system has as many roots outside the unit circle as there are non-predetermined co-state variables. The system thus meets the conditions spelled in Blanchard and Kahn (1980). The solution for the non-predetermined variables and the predetermined state variables are of the form

$$\mathbf{m}_t = \mathbf{A}\mathbf{p}_t + \mathbf{C}\mathbf{z}_t, \tag{12.1}$$

$$\mathbf{p}_{t+1} = \mathbf{B}\mathbf{p}_t + \mathbf{D}\mathbf{z}_t, \tag{12.2}$$

where $\mathbf{m}_t = (\,\widehat{y}_t \quad \widehat{y}_t^* \quad \widehat{n}_t \quad \widehat{n}_t^* \quad \widehat{c}_t \quad \widehat{c}_t^* \quad \widehat{x}_t \quad \widehat{x}_t^*\,)'$ is the vector of non-predetermined variables, $\mathbf{p}_t = (\,\widehat{k}_t \quad \widehat{k}_t^* \quad \widehat{b}_t\,)'$ is the vector of predetermined variables, and $\mathbf{z}_t = (\,\widehat{z}_t \quad \widehat{z}_t^*\,)'$ is the vector of productivity shocks. The transformed variables are of the form $\widehat{a}_t = \ln(a_t/a) \approx (a_t - a)/a$ where $a_t = (\,y_t \quad y_t^* \quad n_t \quad n_t^* \quad c_t \quad c_t^* \quad x_t \quad x_t^* \quad k_t \quad k_t^* \quad z_t \quad z_t^*\,)$, except for $\widehat{b}_t = b_t/y$.

The problem is that the roots of the parameter matrix $\mathbf{B}$ show that the system of predetermined variables (12.2) is not stationary. This implies that the system of non-predetermined variables (12.1) is also non-stationary. Specifically, the roots of $\mathbf{B}$ for the case with the BKK shock process are (0.877, 0.966, 1) and the roots for the case with the BC shock process are (0.936, 0.968, 1). In practice, this implies that deviations from our selected initial state are permanent. This occurs because the deterministic system does not yield a unique solution, and any deviation displaces the system permanently to another solution. Unfortunately, this makes the approximation of the dynamics near the selected steady state questionable.

2.3 The Complete Markets (CM) Model

The non-uniqueness of the deterministic steady state and the associated non-stationarity of the linear solution does not occur in the complete markets version of the two-country IRBC model.

In the CM model, the consumer's budget constraint is

$$c_t + x_t + \int q^w(s_{t+1})b(s_{t+1}|s_t)ds_{t+1} = w_t n_t + r_t^k k_t + b(s_t), \tag{13}$$

where $q^w(s')$ and $b(s'|s)$ are the price and quantity of a state contingent bond purchased in state of the world s that will pay only in the state of the world s' next period. The states of the world follow a stochastic process with transition probability density given by $f(s'|s)$. The asset market clearing conditions are

$$b(s_t) + b^*(s_t) = 0. \tag{14}$$

As in the IM model, the consumer chooses consumption, employment, and capital and bonds holdings to maximize expected lifetime utility (1) subject to the budget constraint (13) and the accumulation equation (4). The competitive firm hires labor and capital to maximize profits (6) subject to the production technology (7). The set of first-order conditions of the consumer's and firm's problems, as well as the asset market clearing conditions form the system of equations that characterizes the symmetric equilibrium.

The system of equations is similar to the system (10). The home and foreign pricing equations (10.3), however, are replaced by

$$q^w(s_{t+1}) = \beta f(s_{t+1}|s_t)\lambda_{t+1}/\lambda_t, \tag{15.1}$$

$$q^w(s_{t+1}) = \beta f(s_{t+1}|s_t)\lambda^*_{t+1}/\lambda^*_t. \tag{15.2}$$

These pricing equations hold for all periods and all states of the world. They thus imply that $\lambda_{t+1}/\lambda_t = \lambda^*_{t+1}/\lambda^*_t$. In the numerical implementation, it is difficult and impractical to compute all prices and holdings. Thus, to simplify the system, we replace the pricing equations by

$$\lambda_t = \lambda^*_t, \tag{16}$$

where we impose that $\lambda_0 = \lambda^*_0$. The steady state of equation (16) is $\lambda = \lambda^*$. We also use the asset market clearing conditions (14) to collapse the home and foreign budget constraints in the goods market clearing condition

$$c_t + c^*_t + x_t + x^*_t = y_t + y^*_t. \tag{17}$$

Then, the equilibrium of the CM model is characterized by home and foreign variants of equations (10.1), (10.2), (10.4), (10.5), and (10.6). Equations (10.3), (10.7), and (10.8) are replaced by equations (16) and (17). The CM model system and its companion steady state system have 12 independent equations and must solve for 12 variables. The steady state is thus unique.

For the numerical implementation, we further simplify the system. To do so, we use equation (10.1) to substitute out λ_t and λ_t^*. We also use the parameter values of the baseline model. Note that the CM model has only 2 state variables. The linear solution is as in equations (12), except that $\mathbf{p}_t = (\,\widehat{k}_t \quad \widehat{k}_t^*\,)'$ is the vector of predetermined variables. The roots of the parameter matrix $\mathbf{B}$ with the BKK process are (0.881, 0.966) and with the BC process are (0.956, 0.971). The linear system is thus stationary.

3. Stationary Incomplete Markets Models

In this section, we present 5 incomplete markets models that yield unique deterministic steady states and stationary linear systems. The different models add assumptions to the consumer's problem to correct the anomalous steady state behavior of the home and foreign pricing equations (10.3).

In all cases, the firm's problem and the asset market clearing condition are as in the IM model. The Technical Appendix presents the system of equations that characterizes the equilibrium for each of the 5 stationary incomplete markets models.

3.1 The Endogenous Discount Factor (DF) Model

The DF model assumes that the consumer's subjective discount factor is endogenous, as in Kollmann (1992) and Corsetti, Dedola, and Leduc (2004). The consumer's expected lifetime utility is

$$E_0\left[\sum_{t=0}^{\infty}\theta_t u(c_t, n_t)\right], \tag{18}$$

where

$$\theta_{t+1} = \beta(c_t, n_t)\theta_t, \tag{19}$$

$$\beta(c_t, n_t) = [1 + c_t^{\eta}(1 - n_t)^{1-\eta}]^{-\zeta}, \tag{20}$$

$\theta_0 = 1$ and $\zeta \geq 0$. Also, β_{ct} and β_{nt} are the derivatives of the discount factor $\beta(c_t, n_t)$ with respect to c_t and n_t.

As before, the consumer chooses consumption, employment, and capital and bond holdings to maximize his expected lifetime utility (18) subject to the budget constraint (3) and the accumulation equation (4). The resulting home and foreign bond pricing equations are

$$q_t^w = \beta_t E_t[\lambda_{t+1}]/\lambda_t, \tag{21.1}$$

$$q_t^w = \beta_t^* E_t[\lambda_{t+1}^*]/\lambda_t^*, \tag{21.2}$$

where $\beta_t = \beta(c_t, n_t)$ and $\beta_t^* = \beta(c_t^*, n_t^*)$. In the deterministic steady state, these equations reduce to two independent equations: $q^w = \beta(c, n)$ and $q^w = \beta(c^*, n^*)$. The deterministic steady state is thus unique.

We implement our numerical linearization method as in the IM model, with one exception. This version of the model replaces the parameter β with the function $\beta(c, n)$, which contains the parameter ζ. We set ζ to ensure that the steady state value of $\beta(c, n) =$ 0.99 as in the IM model. For this version, the roots of the parameter matrix $\mathbf{B}$ for the BKK process are (0.884, 0.959, 0.996). The roots for the BC process are (0.929, 0.962, 0.996). The linear system is thus stationary.

3.2 The Endogenous Discount Factor without Internalization (DFwI) Model

The DFwI model also assumes that the consumer's subjective discount factor is endogenous. The discount factor depends on aggregate consumption and aggregate employment, and the consumer does not internalize the effects of his choices on the discount factor. A similar assumption is used in Schmitt-Grohé and Uribe (2003).

The consumer's expected lifetime utility is as in (18), but the discount factor is given by

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{n}_t)\theta_t, \tag{22}$$

where $\tilde{c}_t$ and $\tilde{n}_t$ are the average per capita consumption and employment in the country. As before, the consumer chooses consumption, employment, and capital and bond holdings to maximize expected lifetime utility (18) subject to the budget constraint (3) and the accumulation equation (4). The resulting home and foreign bond pricing equations are as in equations (21) above, and their deterministic steady state is given by $q^w = \beta(c, n)$ and $q^w = \beta(c^*, n^*)$. The steady state is thus unique.

We implement our numerical linearization method as in the DF Model. The resulting roots of the parameter matrix **B** with the BKK shock process are (0.876, 0.964, 0.996), and the roots with the BC shock process are (0.928, 0.966, 0.996). The linear system is thus stationary.

3.3 The Debt Elastic Interest Rate (DER) Model

The DER model assumes that home and foreign consumers face different prices for the bond, and that the spread between home and foreign prices is a function of the net foreign asset position. A similar assumption appears in Boileau and Normandin (2004) and in Devereux and Smith (2003). Presumably, the spread exists because international financial markets are costly to operate. The consumer's budget constraint is

$$c_t + x_t + q_t b_{t+1} = w_t n_t + r_t^k k_t + b_t, \tag{23}$$

where q_t and q_t^* are the price of the bond faced by the home and foreign consumers. As in Boileau and Normandin (2004), the interest differential is

$$\left(\frac{R_{t+1}^* - R_{t+1}}{R_{t+1} R_{t+1}^*}\right) b_{t+1} = \varphi(b_{t+1}^2 / y_t + b_{t+1}^{*2} / y_t^*), \tag{24}$$

where $R_{t+1} = 1/q_t$, $R_{t+1}^* = 1/q_t^*$, and $\varphi \geq 0$. Equation (24) states that international financial markets charge a higher rate to borrowers than the rate promised to lenders.

The consumer chooses consumption, employment, and capital and bond holdings to maximize expected lifetime utility (1) subject to the budget constraint (23) and the accumulation equation (4). The home and foreign bond pricing equations are

$$q_t = \beta E_t \left[\lambda_{t+1}\right] / \lambda_t, \tag{25.1}$$

$$q_t^* = \beta E_t \left[\lambda_{t+1}^*\right] / \lambda_t^*. \tag{25.2}$$

As required, the deterministic steady state of these equations imply two independent equations: $q = \beta$ and $q^* = \beta$, while the steady state of equation (24) yields $(R^* - R)b = \varphi RR^*(b^2/y + b^{*2}/y^*)$. The steady state is thus unique.

We implement our numerical linearization method as in the IM model. We set the responsiveness of the real interest rate differential to changes in the net foreign asset position to the value found in Lane and Milesi-Ferreti (2002). In the steady state, the responsiveness is φ/β^2 since $R = R^* = 1/\beta$. Thus, we set $\varphi = \beta^2 * 0.01$. The resulting roots of the parameter matrix **B** with the BKK process are (0.600, 0.965, 0.967) and with the BC process are (0.577, 0.965, 0.967). The linear system is thus stationary.

3.4 The Quadratic Portfolio Costs (QPC) Model

The QPC model assumes quadratic portfolio costs, as in Heathcote and Perri (2002). These costs are motivated by small costs to buying the bond. In this case, the consumer's budget constraint is

$$c_t + x_t + q_t^w b_{t+1} + \frac{\pi}{2} b_{t+1}^2 = w_t n_t + r_t^k k_t + b_t, \tag{26}$$

where $\pi \geq 0$.

The consumer chooses consumption, employment, and capital and bond holdings to maximize expected lifetime utility (1) subject to the budget constraint (26) and the accumulation equation (4). The home and foreign bond pricing equations are

$$q_t^w = \beta E_t[\lambda_{t+1}]/\lambda_t - \pi b_{t+1}, \tag{27.1}$$

$$q_t^w = \beta E_t[\lambda_{t+1}^*]/\lambda_t^* - \pi b_{t+1}^*. \tag{27.2}$$

The deterministic steady state of equations (27) yields two independent equations: $q^w = \beta - \pi b$ and $q^w = \beta - \pi b^*$. The steady state is thus unique.

We implement our numerical linearization method as in the IM model. We set $\pi = \beta^2 * 0.01$ to ensure that the QPC model is comparable to the DER model. The resulting roots of the parameter matrix **B** with the BKK process are (0.578, 0.965, 0.967) and with the BC process are (0.233, 0.965, 0.967). The linear system is thus stationary.

3.5 The Direct Preferences for Wealth (DPW) Model

The DPW model assumes that consumers care about their relative wealth, as in Gong and Zou (2002) and Fisher and Hof (2005). The consumer's expected lifetime utility is

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \mathrm{v}\big(c_t, n_t, v_t\big)\right], \tag{28}$$

where $v_t = (k_t + b_t)/v_t^w$ is an index of status, $v_t^w = k_t + b_t + k_t^* + b_t^*$ is a reference status point,

$$\mathrm{v}(c_t, n_t, v_t) = u(c_t, n_t)v_t^{-\chi}, \tag{29}$$

and $\chi \geq 0$.

The consumer chooses consumption, employment, and capital and bond holdings to maximize expected lifetime utility (28) subject to the budget constraint (3) and the accumulation equation (4), and the consumer takes the reference status as given. The home and foreign bond pricing equations are

$$q_t^w = \beta E_t\left[\lambda_{t+1} + \mathrm{v}_{vt+1}/v_{t+1}^w\right]/\lambda_t, \tag{30.1}$$

$$q_t^w = \beta E_t\left[\lambda_{t+1}^* + \mathrm{v}_{vt+1}^*/v_{t+1}^w\right]/\lambda_t^*, \tag{30.2}$$

where v_{vt} is the partial derivative of $\mathrm{v}(c_t, n_t, v_t)$ with respect to v_t. The deterministic steady state of these equations imply two independent equations: $q^w = \beta[1 + \mathrm{v}_v/(\lambda v^w)]$ and $q^w = \beta[1 + \mathrm{v}_v^*/(\lambda^* v^w)]$. The steady state is unique.

We again implement our numerical linearization method as in the IM model. This version adds the parameter χ. The evidence is scant on the value of this parameter. Bakshi and Chen (1996) find empirical evidence for χ to be somewhere between 0.75 and 1.27. With our current calibration, these values however imply a highly negative rate of return. Instead, we use a small value of $\chi = 0.1$ to avoid large negative rates of return. The resulting roots of the parameter matrix **B** with the BKK process are (0.867, 0.970, 0.976) and with the BC process are (0.936, 0.971, 0.975). The linear system is thus stationary.

4. Numerical Results

Baxter and Crucini (1995) argue that the IM model generates a differential wealth effect that explains the lack of international risk sharing found in the data and that solves the quantity anomaly. This effect, however, is sensitive to the parametrization of the shock process. It occurs only when shocks are highly persistent and do not spill over international boundaries.

In this section, we verify whether the differential wealth effect and the ability to solve the quantity anomaly survive in stationary incomplete markets IRBC models. For this, we compare the business cycle moments and impulse responses of the different models driven by both BKK and BC parametrizations of the shock process.

4.1 Business Cycle Moments and Impulse Responses

We first compare a number of business cycle moments and impulse responses generated by the various models for the two different parametrizations of the shock process. Table 1 and Figure 1 present the moments and responses for the models driven by the BKK shock process. Table 2 and Figure 2 present the same information for the models driven by the BC shock process. The moments are the standard deviations of a variable relative to that of the logarithm of output, the within-country correlation between a variable and the logarithm of output, and the cross-country correlation between variables. The variables are the logarithms of output, consumption, investment, and employment, as well as the net exports to output ratio. The empirical moments are taken from Backus, Kehoe, and Kydland (1995). The predicted moments are computed as the averages of 1000 replications of 200 periods. As in Hodrick and Prescott (1997), all variables are detrended using the Hodrick-Prescott filter. Note that this permit the computation of second order moments from the (non-stationary) IM model. The impulse responses are the percentage responses of output, consumption, investment, and net exports to output ratio following a one standard deviation positive shock to home productivity generated by the unfiltered model.

Note that the stationary incomplete markets models can be made arbitrary close to the IM model. To show this, the following parameters must be changed. First, for the DF and DFwI models, the exercise requires that the discount factor be restated as $\beta(c_t, n_t) = \bar{\beta}[1 + c_t^{\eta}(1 - n_t)^{1-\eta}]^{-\zeta}$, where $\bar{\beta}$ is calibrated to ensure that $\beta(c, n) = 0.99$ in the deterministic steady state. Then, the predictions of the stationary models converge to those of the IM model as $\zeta \to 0$ for the DF and DFwI models, as $\varphi \to 0$ for the DER model, as $\pi \to 0$ for the QPC model, and as $\chi \to 0$ for the DPW model. In fact, for small enough values of these parameters, the stationary models produce moments and responses that are almost identical to those of the IM model.

Table 1 and Figure 1 show that the IM model and the stationary models yield somewhat similar business cycle moments and impulse responses when driven by the BKK shock process. First, the moments and responses are very similar to those of the CM model. Second, Table 1 suggests that the different models replicate the data fairly well. In particular, they correctly predict that consumption, employment, and the net exports to output ratio are less volatile than output. They also correctly predict that consumption, investment, and employment are procyclical, while the net exports to output ratio is countercyclical. The main discrepancy between the predictions and the data is that the predicted cross-country correlation of consumption is incorrectly larger than that of output for all models, leading to the quantity anomaly. Other discrepancies include that all models understate the relative volatility of consumption and employment, as well as the persistence of output. Also, the DPW model grossly overstates the volatility of the net exports to output ratio, while the CM, IM, DF, and DFwI models understate the extent to which the net exports to output ratio is countercyclical. The dynamic responses are all very similar, but the DPW model produces large fluctuations of the net exports to output ratio.

Table 2 and Figure 2 show that the models driven by the BC shock process no longer yield similar business cycle moments and impulse responses. First, the moments and responses computed from the incomplete markets models differ markedly from those of the CM model. Importantly, the incomplete markets models all generate a larger cross-country correlation for output than for consumption, and thus resolve the quantity anomaly. The CM model, however, does not. Second, the moments of the incomplete markets models differ in some crucial ways. The business cycle moments generated by the DF, DFwI, and DPW models are far from those of the data. The DF and DFwI models generate too much volatility for consumption and for the net export to output ratio. In addition, they grossly understate the extent to which employment is procyclical. The DPW model generates too much volatility for the net export to output ratio. Finally, only the DER and QPC models produce a positive cross-country correlation of consumption. In fact, for these two models, the ratio of the cross-country correlations of consumption and output is roughly 77 percent, as in the data. Third, the responses of the IM and stationary models also differ. The responses of output and consumption appear similar across the different models, but the responses of investment, employment, and the net exports to output ratio differ considerably. Notably, the DF and DFwI models predict a reduction of employment following the positive productivity shock, and the DPW model generates large fluctuations for the net export to output ratio.

Overall, the ability to solve the quantity anomaly is shared by all incomplete financial markets models driven by the BC shock process. In this case, however, the models predict different moments and responses. Importantly, the DF, DFwI, and DPW models produce counterfactual business cycle moments. Also, the DER and QPC models generate nearly identical moments and responses, and replicate the observed ratio of the cross-country correlations of consumption and output.

4.2 Wealth Effects in Incomplete Markets Models

The baseline and stationary incomplete markets models driven by the BKK shock process mimic the complete markets model and thus fail to solve the quantity anomaly. In contrast, the incomplete markets models driven by the BC process do not mimic the complete markets model and solve the quantity anomaly. This suggests that the one-period bond is a good financial instrument to share risk when productivity shocks are not highly persistent (ρ small) and rapidly spill over international boundaries (ν large). The one-period bond, however, is not a good instrument to share risk when productivity shocks are highly persistent (ρ large) and do not spill over international boundaries (ν small).

Baxter and Crucini (1995) argue that this occurs because of a differential wealth effect. They reach this conclusion using King's (1990) Hisckian decomposition of the responses of consumption and employment into wealth and price effects (wage and interest rate). The decomposition is computed as follows. Consider the responses of consumers to a positive innovation to home productivity, as shown in Figure 1 and 2. First, for given prices, consumers alter their consumption and employment choices because the higher productivity changes their wealth. The wealth effect is measured as the constant responses of consumption and employment that produce a change in the lifetime utility identical to that produced by the home productivity shock, holding prices constant. Second, for given wealth, consumers also alter their choices because the higher productivity changes both the wage rate and the interest rate that they face. The wage rate effect is measured as the responses that result from the changes in the wage rate due to the higher home productivity, holding wealth and interest rate constant. The interest rate effect is measured as the responses that result from the changes in the interest rate, holding wealth and wage rate constant.

Figures 3 to 5 show the decompositions of the home and foreign responses of consumption and employment to a positive innovation to home productivity. The figures show the decompositions for the CM, IM, and DER models driven by the BC shock process. We do not present the other stationary models for two reasons. First, as stated previously, the DER and QPC models outperform the other stationary models. Second, these models generate identical business cycle moments, because we have parametrized the QPC model to mimic the DER model. Finally, we focus on the BC shock process because it helps solve the quantity anomaly.

A comparison between the figures suggests that the IM and DER models generate very similar wealth effects, but the CM model does not. Figure 3 shows that the home productivity shock in the CM model raises home and foreign consumption. It also raises home employment, but reduces foreign employment as productive resources are reallocated to the home economy. With our calibration, the result is a small positive wealth effect at home, and a much larger positive wealth effect abroad. The home wealth effect slightly raises consumption and slightly reduces employment. The foreign wealth effect largely raises consumption and reduces employment. Figures 4 and 5 show that the home productivity shock in the IM and DER models results in a large positive wealth effect at home, and a much smaller positive wealth effect abroad. This largely raises home consumption and reduces home employment. It also slightly raises foreign consumption and reduces foreign employment.

As in Baxter and Crucini (1995), the differences in the wealth effects explains why incomplete markets resolve the quantity anomaly. Under complete markets, the rise in home productivity generates an increase in home output that must be shared abroad. The sharing reduces the wealth effect at home while raising the wealth effect abroad. Under incomplete markets, the rise in home output needs not be fully shared abroad. The lesser extent of sharing explains a much larger wealth effect at home, and a smaller wealth effect abroad. The difference in the home and foreign wealth effects unties home and foreign consumption, and reduces the cross-country correlation of consumption. The wealth effects, however, raise employment at home and abroad, which raises the cross-country correlation of output.

In addition, we find that the decompositions for the IM and DER models differ in subtle ways. First, the DER model produces an interest rate effect that lowers foreign consumption less than in the IM model, and thus explains a larger increase in foreign consumption. Second, the DER model produces an interest rate effect that lowers home consumption more than in the IM model. It also produces a wage effect that raises home consumption less than in the IM model. The joint effects explains why home consumption does not rise as much as in the IM model. The result of the price effects for the DER model is that home consumption rises less and foreign consumption more in response to a home productivity shock. This promotes a positive cross-country consumption correlation in the DER model. Finally, the price effects also explain why home employment responds more than foreign employment in the DER model, compared to the IM model.

Overall, the IM and DER models driven by the BC shock process generate large wealth effects at home and small wealth effects abroad. The models, however, generate different interest rate and wage effects. These price effects explain the differences between the predictions of the two models, especially for the cross-country correlations of consumption and output.

4.3 Extensions

The Hicksian decompositions document that the difference between the baseline IM model and the DER model lies in the price effects. This suggests that the general equilibrium responses of wages and interest rates are important to understand the differences between the incomplete markets models. For this reason, we extend our analysis to study the effects of restrictions to the labor and capital markets.

Table 3 and Figures 6 and 7 show business cycle moments and wealth decompositions for two versions of the IM and DER models. The No N version is aimed at the labor market. It assumes that consumers supply labor inelastically. Accordingly, employment does not respond to changes in productivity, and stays at its deterministic steady state value of $n = 0.3$. This obviously eliminates any wealth and price effects on employment. The No K version is aimed at the capital market. It assumes an inelastic supply of physical capital. This eliminates the responses of investment and capital to changes in productivity. For this, we set the depreciation rate to $\delta = 0$ and investment is forced to remain at a steady state of $x = 0$.

The results indicate that that the IM and DER models yield different business cycle moments and decompositions. First, the IM and DER models solve the quantity anomaly for the No N version, but not for the No K version. Second, the models generate some important discrepancies with the data. By construction, the No N versions eliminate the fluctuations of employment, while the No K versions eliminate the fluctuations of investment. The No K assumption also seriously dampens the volatility of employment and of the net exports to output ratio, and makes the net exports to output ratio procyclical. Third, as before, the different versions of the two models produce a larger wealth effect at home than abroad for consumption, although much less so for the inelastic capital versions. Fourth, in the No N versions, the wage and interest rate effects almost cancel each other, so that most of the total effect on consumption is due to the wealth effect. Fifth, in the No K versions, the absence of capital eliminates the interest rate effect.

Overall, these results suggest that the responses of employment are not the only contributor to the solution of the quantity anomaly. The responses of physical capital play an important role in explaining the cross-country correlations of consumption and output.

This occurs because of the price effects, and especially of the interest rate effect.

5. Conclusion

Several authors argue that the baseline IRBC model with incomplete international financial markets provides a solution to the *quantity anomaly*. For this, productivity shocks must be highly persistent and must not spill over international boundaries.

Unfortunately, the above conclusion is suspect because it stems from an analysis of the near steady state dynamics using a linearized system of equations. The baseline IRBC model with incomplete financial markets does not possess a unique deterministic steady state and, as a result, its linear system of difference equations is not stationary.

We show that the ability to solve the quantity anomaly is robust to modifications of the model that ensure the existence of a unique steady state and a stationarity system of linear difference equations. We find, however, that the modifications affect the quantitative predictions regarding key macroeconomic variables, especially when the model solves the quantity anomaly.

References

Backus, D.K., P.J. Kehoe, and F.E. Kydland, 1995, International business cycles: Theory and evidence, in T. Cooley (ed.) *Frontiers of Business Cycle Research*, Princeton: Princeton University Press.

Backus, D.K., P.J. Kehoe, and F.E. Kydland, 1992, International real business cycles, *Journal of Political Economy* **101**, 745–775.

Bakshi, G.S. and Z. Chen, 1996, The spirit of capitalism and stock-market prices, *American Economic Review* **86**, 133–157.

Baxter, M. and M.J. Crucini, 1995, Business cycles and the asset structure of foreign trade, *International Economic Review* **36**, 821–854.

Blanchard, O. and C.M. Kahn, 1980, The solution to linear difference models under rational expectations, *Econometrica* **48**, 1305–1313.

Boileau, M. and M. Normandin, 2004, Dynamics of the current account and interest differentials, HEC Montréal Working Paper 2004-01.

Corsetti, G., L. Dedola, and S. Leduc, 2004, International risk sharing and the transmission of productivity shocks, European Central Bank Working Paper Series No. 308.

Devereux, M.B. and G.W. Smith, 2003, Transfer problem dynamics: Macroeconomics of the Franco-Prussian war indemnity, mimeo Queen's University.

Fisher, W. and F.X. Hof, 2005, Status seeking in the small open economy, forthcoming *Journal of Macroeconomics*.

Gong, L. and H. Zou, 2002, Direct preferences for wealth, the risk premium puzzle, growth, and policy effectiveness, *Journal of Economic Dynamics and Control* **26**, 271–302.

Heathcote, J. and F. Peri, 2002, Financial autarky and international business cycles, *Journal of Monetary Economics* **49**, 601–627.

Hodrick, R.J. and E.C. Prescott, 1997, Postwar U.S. business cycles: An empirical investigation, *Journal of Money, Credit and Banking* **29**, 1–16.

Kehoe, P. and F. Perri, 2002, International business cycles with endogenous incomplete markets, *Econometrica* **70**, 907–928.

Kim, J., S. Kim, and A. Levin, 2003, Patience, persistence and properties of two-country incomplete market models, *Journal of International Economics* **61**, 385–396.

Kim, S. and A. Kose, 2003, Dynamics of open economy business cycle models: Understanding the role of the discount factor, *Macroeconomic Dynamics* **7**, 263–290.

King, R.G., 1990, Value and capital in the equilibrium business cycle program, in L. McKenzie and S. Zamagni (eds.) *Value and Capital: Fifty Years Later, London: MacMillan.*

King, R.G., Plosser, C.I., and S.T. Rebelo, 2002, Production, growth and business cycles: Technical appendix, *Computational Economics* **20**, 87–116.

Kollmann, R., 1996, Incomplete asset markets and the cross-country consumption correlation puzzle, *Journal of Economic Dynamics and Control* **20**, 945–961.

Kollmann, R., 1992, Incomplete asset markets and the cross-country consumption correlation puzzle, mimeo University of Chicago.

Lane, P.R. and G.M. Milesi-Ferretti, 2002, Long term capital movements, in B.S. Bernanke and K. Rogoff (eds.) *NBER Macroeconomics Annual 2001*, Cambridge: MIT Press.

Schmitt-Grohé, S. and M. Uribe, 2003, Closing small open economies, *Journal of International Economics* **61**, 163–185.

A. Technical Appendix

In this appendix, we present the system of equations that characterizes the equilibrium for each stationary incomplete markets model.

A.1 The Endogenous Discount Factor (DF) Model

The system of equations that characterizes the equilibrium of the DF model includes home and foreign variants of

$$\lambda_t = u_{ct} - \beta_{ct} a_t, \tag{A1.1}$$

$$u_{nt} = -\lambda_t (1-\alpha) y_t / n_t + \beta_{nt} a_t, , \tag{A1.2}$$

$$q_t^w = \beta_t E_t \left[\lambda_{t+1}\right] / \lambda_t, \tag{A1.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta_t E_t \left[\lambda_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}} \left(\phi_{t+1} - \phi_{\delta t+1} \frac{x_{t+1}}{k_{t+1}} + (1-\delta) \right) \right], \tag{A1.4}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{A1.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta) k_t, \tag{A1.6}$$

$$y_t = c_t + x_t + q_t^w b_{t+1} - b_t, \tag{A1.7}$$

$$a_t = -E_t[u_{t+1}] + E_t[\beta_{t+1} a_{t+1}], \tag{A1.8}$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0, \tag{A1.9}$$

where $u_t = u(c_t, n_t)$. The system (A1) has 17 independent equations and must solve 17 variables. The two additional endogenous variables are a_t and a_t^*. The deterministic steady state of system (A1) also has 17 independent equations that solve for 17 variables. That is, the deterministic steady state is unique.

A.2 The Endogenous Discount Factor without Internalization (DFwI) Model

The system of equations that characterizes the equilibrium of the DFwI model includes home and foreign variants of

$$\lambda_t = u_{ct}, \tag{A2.1}$$

$$u_{nt} = -\lambda_t (1-\alpha) y_t / n_t, \tag{A2.2}$$

$$q_t^w = \beta_t E_t \left[\lambda_{t+1}\right] / \lambda_t, \tag{A2.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta_t E_t \left[\lambda_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}} \left(\phi_{t+1} - \phi_{\delta t+1} \frac{x_{t+1}}{k_{t+1}} + (1-\delta) \right) \right], \tag{A2.4}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{A2.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta) k_t, \tag{A2.6}$$

$$y_t = c_t + x_t + q_t^w b_{t+1} - b_t, \tag{A2.7}$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0, \tag{A2.8}$$

where $\tilde{c}_t = c_t$ and $\tilde{n}_t = n_t$. The system (A2) and its companion deterministic steady state system both have 15 independent equations and must solve for 15 variables. The solution to the steady state is unique.

A.3 The Debt Elastic Interest Rate (DER) Model

The system of equations that characterizes the equilibrium of the DER model includes home and foreign variants of

$$\lambda_t = u_{ct}, \tag{A3.1}$$

$$u_{nt} = -\lambda_t(1-\alpha)y_t/n_t, \tag{A3.2}$$

$$q_t = \beta E_t\left[\lambda_{t+1}\right]/\lambda_t, \tag{A3.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta E_t\left[\lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}}\left(\phi_{t+1} - \phi_{\delta t+1}\frac{x_{t+1}}{k_{t+1}} + (1-\delta)\right)\right], \tag{A3.4}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{A3.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta)k_t, \tag{A3.6}$$

$$y_t = c_t + x_t + q_t b_{t+1} - b_t, \tag{A3.7}$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0 \tag{A3.8}$$

and the differential

$$\left(\frac{R_{t+1}^* - R_{t+1}}{R_{t+1}R_{t+1}^*}\right)b_{t+1} = \varphi(b_{t+1}^2/y_t + b_{t+1}^{*2}/y_t^*), \tag{A3.9}$$

where $R_{t+1} = 1/q_t$ and $R_{t+1}^* = 1/q_t^*$. The system (A3) and its companion steady state system both have 16 independent equations and must solve 16 variables. The variables include the domestic and foreign asset prices q_t and q_t^*, but not the world asset price q_t^w. Thus, the deterministic steady state is unique.

A.4 The Quadratic Portfolio Costs (QPC) Model

The system of equations that characterizes the equilibrium of the QPC model includes home and foreign variants of

$$\lambda_t = u_{ct}, \tag{A4.1}$$

$$u_{nt} = -\lambda_t(1-\alpha)y_t/n_t, \tag{A4.2}$$

$$q_t^w = \beta E_t\left[\lambda_{t+1}\right]/\lambda_t - \pi b_{t+1}, \tag{A4.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta E_t\left[\lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}}\left(\phi_{t+1} - \phi_{\delta t+1}\frac{x_{t+1}}{k_{t+1}} + (1-\delta)\right)\right], \tag{A4.4}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{A4.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta)k_t, \tag{A4.6}$$

$$y_t = c_t + x_t + q_t^w b_{t+1} + \frac{\pi}{2}b_{t+1}^2 - b_t, \tag{A4.7}$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0. \tag{A4.8}$$

The system (A4) and its companion deterministic steady state both have 15 independent equations and must solve 15 variables. Thus the deterministic steady state is unique.

A.5 The Direct Preferences for Wealth (DPW) Model

The system of equations that characterizes the equilibrium of the DPW model includes home and foreign variants of

$$\lambda_t = \mathrm{v}_{ct}, \tag{A5.1}$$
$$\mathrm{v}_{nt} = -\lambda_t (1-\alpha) y_t / n_t, \tag{A5.2}$$
$$q_t^w = \beta E_t \left[\lambda_{t+1} + \mathrm{v}_{vt+1} / v_{t+1}^w \right] / \lambda_t, \tag{A5.3}$$
$$\frac{\lambda_t}{\phi_{\delta t}} = \beta E_t \left[\lambda_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}} \left(\phi_{t+1} - \phi_{\delta t+1} \frac{x_{t+1}}{k_{t+1}} + (1-\delta) \right) + \frac{\mathrm{v}_{vt+1}}{v_{t+1}^w} \right], \tag{A5.4}$$
$$y_t = z_t k_t^\alpha n_t^{1-\alpha}, \tag{A5.5}$$
$$k_{t+1} = \phi_t k_t + (1-\delta) k_t, \tag{A5.6}$$
$$y_t = c_t + x_t + q_t^w b_{t+1} - b_t, \tag{A5.7}$$
$$v_t = (k_t + b_t) / v_t^w, \tag{A5.8}$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0 \tag{A5.9}$$

and the reference point

$$v_t^w = k_t + b_t + k_t^* + b_t^*, \tag{A5.10}$$

where v_{ct} and v_{nt} are the derivatives of $\mathrm{v}(c_t, n_t, v_t)$ with respect to c_t and n_t. The system (A5) and its companion steady state system both have 18 independent equations and must solve 18 variables. The added variables are v_t, v_t^*, and v_t^w. Thus, the deterministic steady state is unique.

Table 1. Business Cycle Moments with BKK Shock Process

	Data	CM	IM	DF	DFwI	DER	QPC	DPW
Standard deviations relative to output:								
Consumption	0.75	0.45	0.46	0.42	0.48	0.49	0.49	0.43
Investment	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27
Employment	0.61	0.48	0.46	0.50	0.45	0.41	0.41	0.44
Net exports/output	0.27	0.31	0.32	0.32	0.33	0.29	0.29	0.76
Correlations with output:								
Past output	0.86	0.69	0.69	0.69	0.69	0.69	0.69	0.69
Consumption	0.82	0.82	0.85	0.79	0.85	0.92	0.92	0.92
Investment	0.94	0.90	0.89	0.89	0.88	0.90	0.90	0.83
Employment	0.88	0.93	0.93	0.93	0.92	0.94	0.94	0.96
Net exports/output	-0.37	-0.09	-0.11	-0.01	-0.12	-0.30	-0.30	-0.44
Cross-country correlations:								
Output	0.66	-0.03	-0.01	-0.04	-0.03	0.07	0.07	0.07
Consumption	0.51	0.91	0.87	0.94	0.87	0.72	0.72	0.73

Note: Entries under standard deviations relative to output are the ratio of the standard deviation of a variable to that of the logarithm of output. Entries under correlations with output are the contemporaneous correlation between a variable and the logarithm of output. Entries under cross-country correlations are the contemporaneous correlation between home and foreign variables. The variables are the logarithm of output, the logarithm of consumption, the logarithm of investment, the logarithm of employment, and the ratio of net exports and output. All variables are detrended with the Hodrick-Prescott filter. The Data column is taken from Backus, Kehoe, and Kydland (1995), and it refers to U.S. data and U.S. and Europe data for the period 1970:I to 1990:II. Also, CM stands for the complete markets model, IM for the baseline incomplete markets model, DF for the endogenous discount factor model, DFwI for the endogenous discount factor without internalization model, DER for the debt elastic interest rate model, QPC for the quadratic portfolio costs model, and DWP for the direct preferences for wealth model.

Table 2. Business Cycle Moments with BC Shock Process

	Data	CM	IM	DF	DFwI	DER	QPC	DPW
Standard deviations relative to output:								
Consumption	0.75	0.52	0.96	1.11	1.11	0.70	0.70	0.78
Investment	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27
Employment	0.61	0.47	0.25	0.43	0.37	0.22	0.22	0.23
Net exports/output	0.27	0.24	0.82	1.03	1.00	0.57	0.57	1.06
Correlations with output:								
Past output	0.86	0.72	0.72	0.72	0.72	0.72	0.72	0.72
Consumption	0.82	0.78	0.93	0.84	0.88	0.99	0.99	0.96
Investment	0.94	0.90	0.81	0.77	0.76	0.78	0.78	0.79
Employment	0.88	0.88	0.30	0.12	0.04	0.98	0.98	0.77
Net exports/output	-0.37	-0.24	-0.42	-0.33	-0.36	-0.30	-0.30	-0.46
Cross-country correlations:								
Output	0.66	-0.15	0.48	0.65	0.60	0.22	0.22	0.39
Consumption	0.51	0.92	-0.13	-0.32	-0.24	0.17	0.17	-0.06

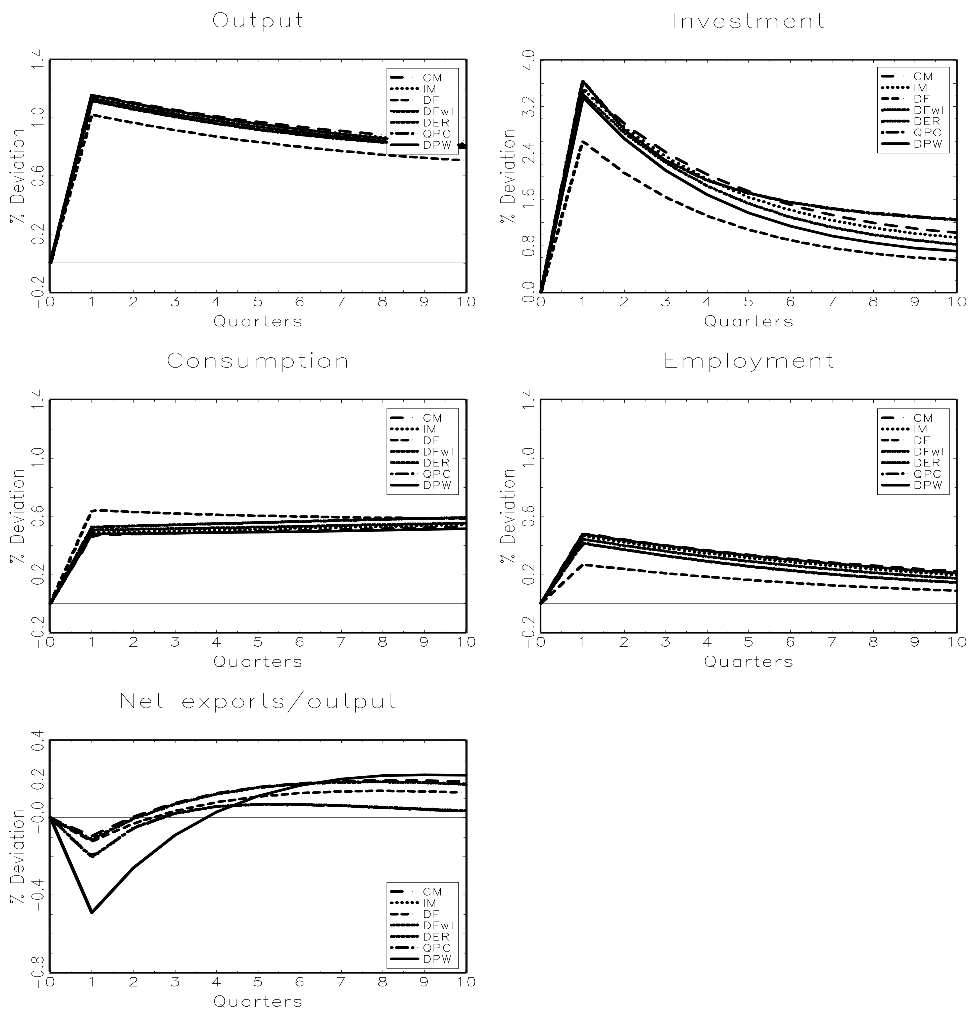
Note: Entries under standard deviations relative to output are the ratio of the standard deviation of a variable to that of the logarithm of output. Entries under correlations with output are the contemporaneous correlation between a variable and the logarithm of output. Entries under cross-country correlations are the contemporaneous correlation between home and foreign variables. The variables are the logarithm of output, the logarithm of consumption, the logarithm of investment, the logarithm of employment, and the ratio of net exports and output. All variables are detrended with the Hodrick-Prescott filter. The Data column is taken from Backus, Kehoe, and Kydland (1995), and it refers to U.S. data and U.S. and Europe data for the period 1970:I to 1990:II. Also, CM stands for the complete markets model, IM for the baseline incomplete markets model, DF for the endogenous discount factor model, DFwI for the endogenous discount factor without internalization model, DER for the debt elastic interest rate model, QPC for the quadratic portfolio costs model, and DWP for the direct preferences for wealth model.

Table 3. Moments in Alternative Models with BC shock process

		IM		DER	
	Data	No N	No K	No N	No K
Standard deviations relative to output:					
Consumption	0.75	0.94	0.95	0.72	1.00
Investment	3.27	3.27	0.00	3.27	0.00
Employment	0.61	0.00	0.06	0.00	0.01
Net exports/output	0.27	0.72	0.08	0.57	0.01
Correlations with output:					
Past output	0.86	0.72	0.70	0.72	0.70
Consumption	0.82	0.97	1.00	0.99	1.00
Investment	0.94	0.86	0.00	0.78	0.00
Employment	0.88	0.00	0.63	0.00	0.57
Net exports/output	-0.37	-0.55	0.63	-0.33	0.57
Cross-country correlations:					
Output	0.66	0.22	0.20	0.21	0.25
Consumption	0.51	-0.23	0.32	0.16	0.26

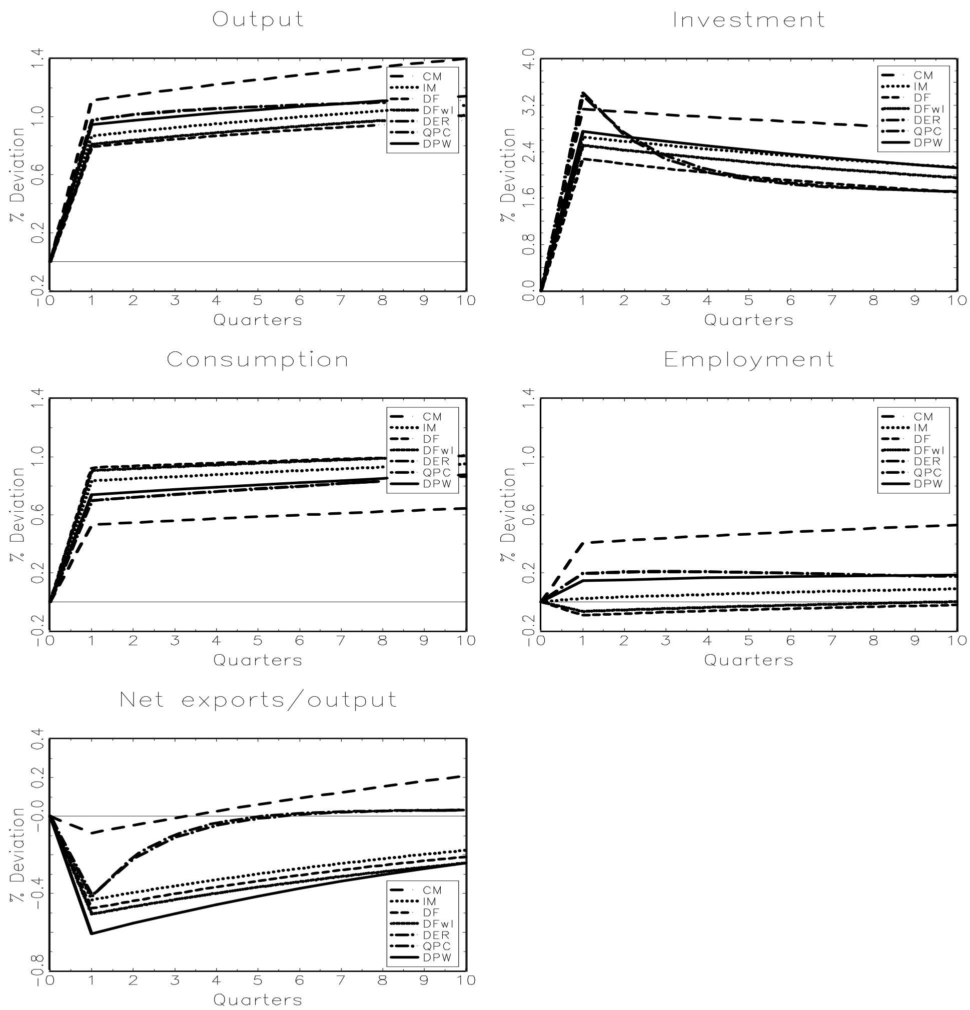
Note: Entries under standard deviations relative to output are the ratio of the standard deviation of a variable to that of the logarithm of output. Entries under correlations with output are the contemporaneous correlation between a variable and the logarithm of output. Entries under cross-country correlations are the contemporaneous correlation between home and foreign variables. The variables are the logarithm of output, the logarithm of consumption, the logarithm of investment, the logarithm of employment, and the ratio of net exports and output. All variables are detrended with the Hodrick-Prescott filter. The Data column is taken from Backus, Kehoe, and Kydland (1995), and it refers to U.S. data and U.S. and Europe data for the period 1970:I to 1990:II. Also, IM stands for the baseline incomplete markets model and DER for the debt elastic interest rate model. Under both IM and DER, No N stands for inelastic labor and No K for inelastic capital.

Figure 1. Dynamic Responses with BKK Shock Process



Note: The figure shows dynamic responses to a one-standard deviation positive shock to home productivity. CM stands for the complete markets model, IM for the baseline incomplete markets model, DF for the endogenous discount factor model, DFwI for the endogenous discount factor without internalization model, DER for the debt elastic interest rate model, QPC for the quadratic portfolio costs model, and DWP for the direct preferences for wealth model.

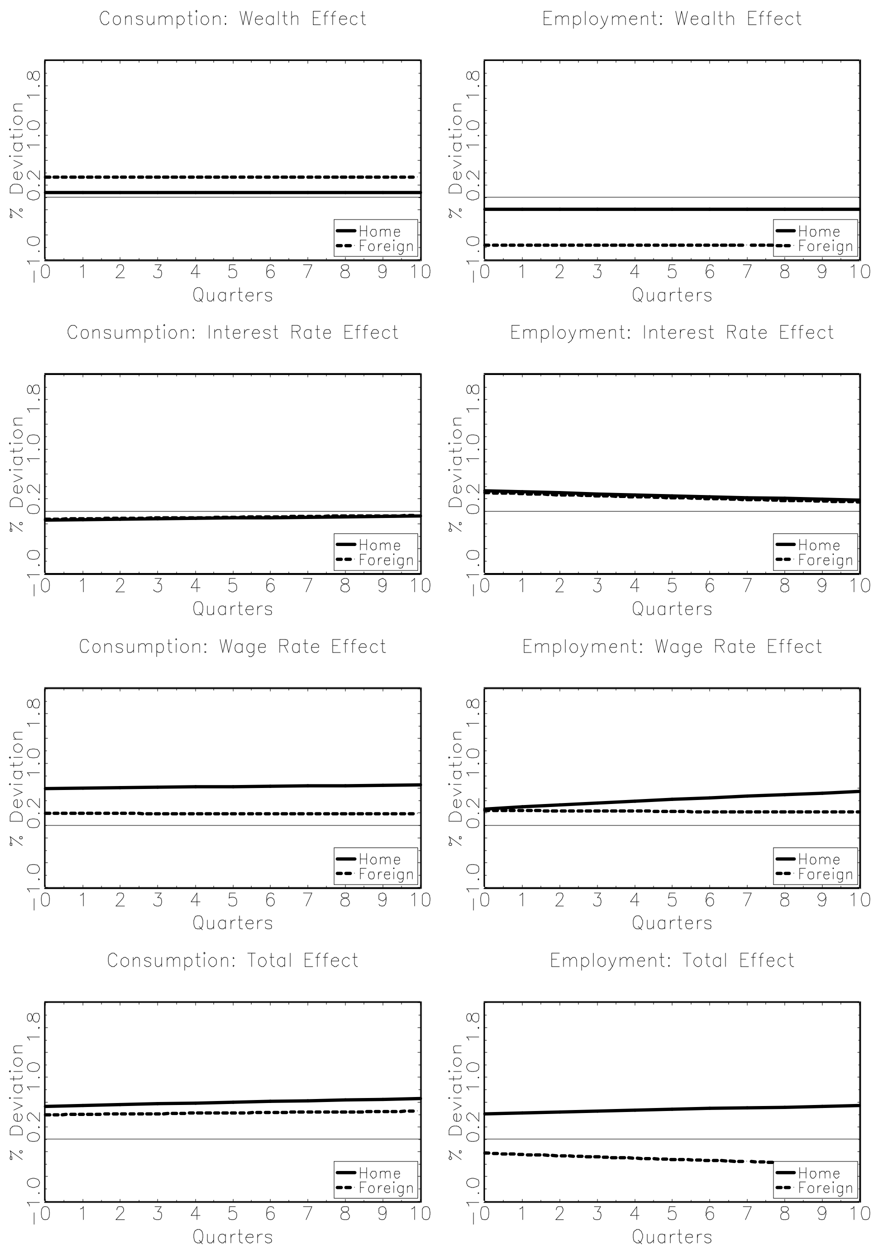
Figure 2. Dynamic Responses with BC Shock Process



Note: The figure shows dynamic responses to a one-standard deviation positive shock to home productivity. CM stands for the complete markets model, IM for the baseline incomplete markets model, DF for the endogenous discount factor model, DFwI for the endogenous discount factor without internalization model, DER for the debt elastic interest rate model, QPC for the quadratic portfolio costs model, and DWP for the direct preferences for wealth model.

Figure 3. Decomposition of Consumption and Employment Responses

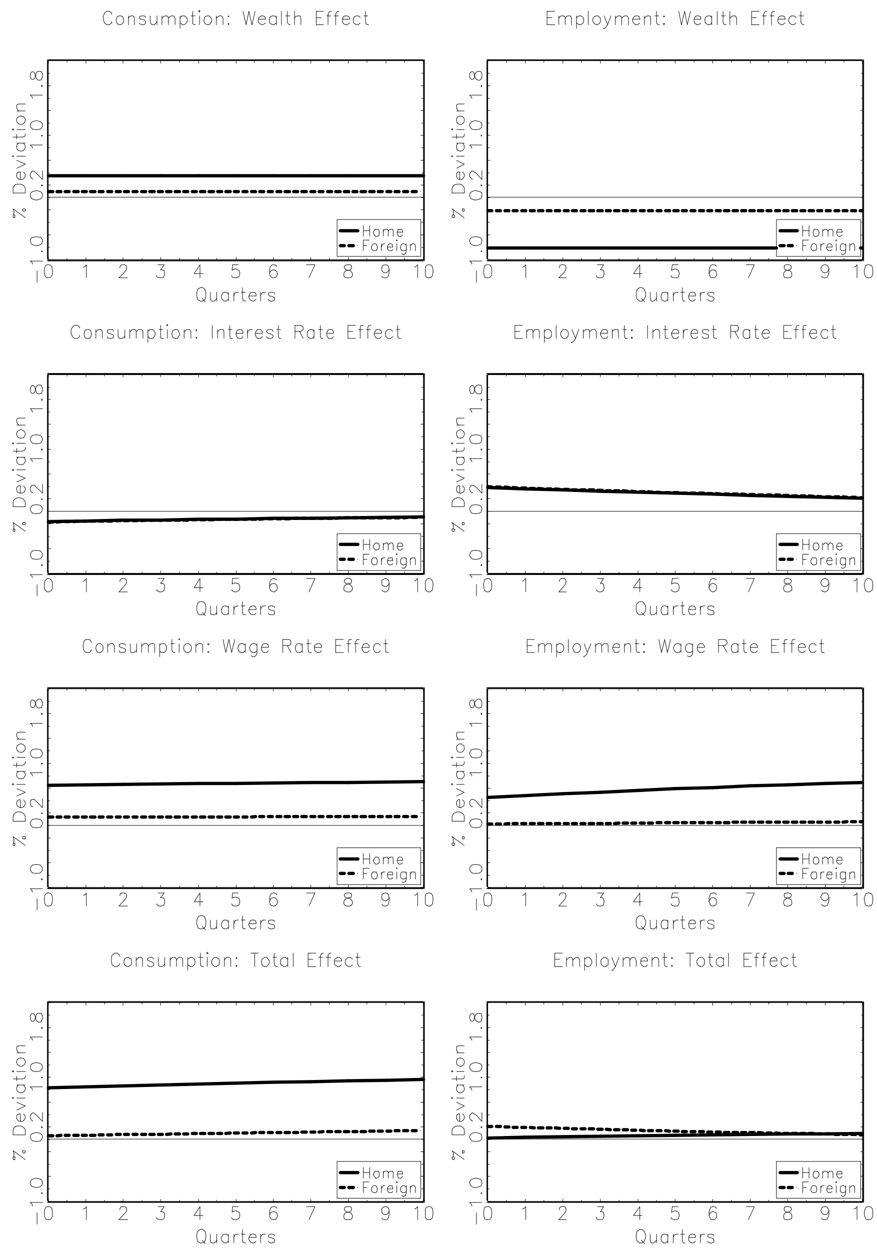
CM with BC Shock Process



Note: The figure shows the Hicksian decomposition of the responses of consumption and employment to a one standard deviation positive shock to home productivity. The responses are computed from the complete markets model with the BC parametrization of the shock process.

Figure 4. Decomposition of Consumption and Employment Responses

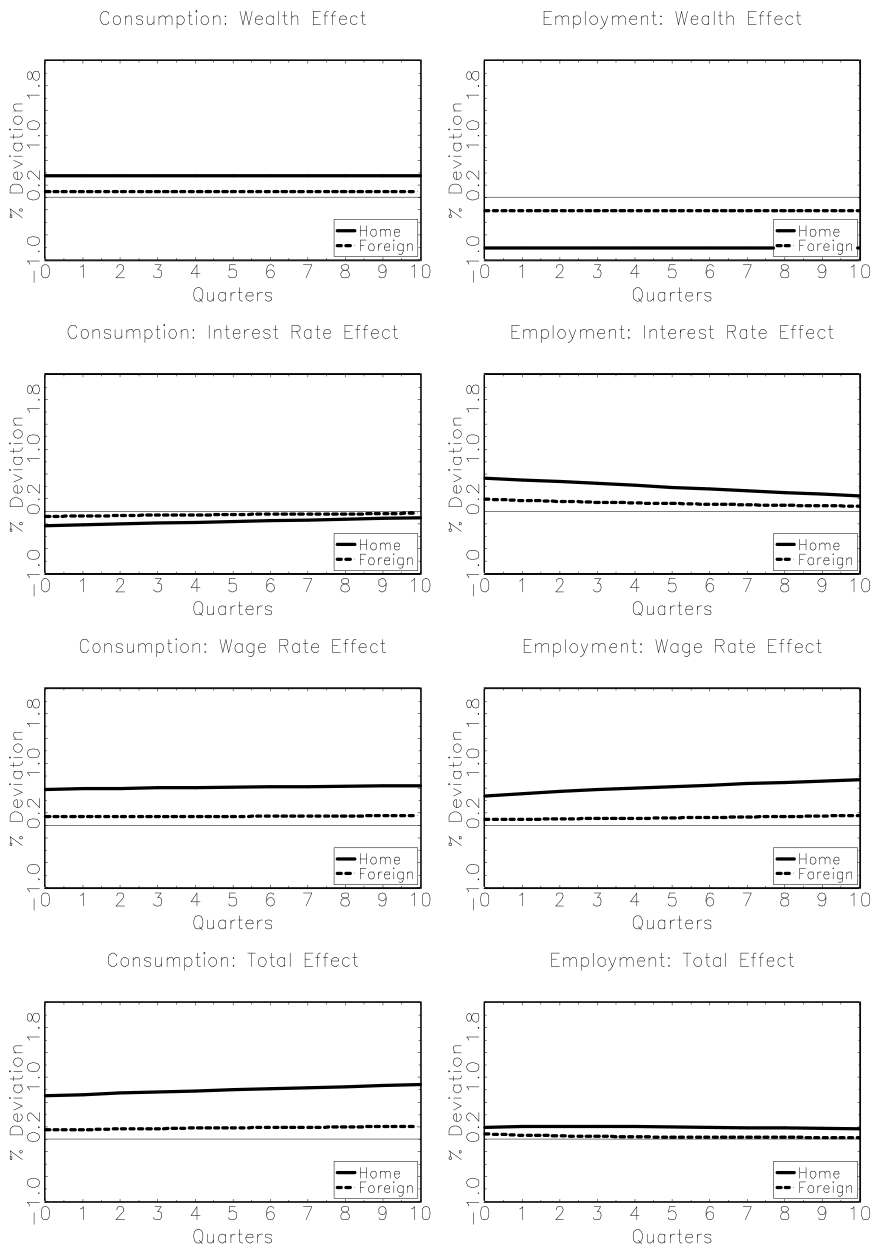
IM with BC Shock Process



Note: The figure shows the Hicksian decomposition of the responses of consumption and employment to a one standard deviation positive shock to home productivity. The responses are computed from the baseline incomplete markets model with the BC parametrization of the shock process.

Figure 5. Decomposition of Consumption and Employment Responses

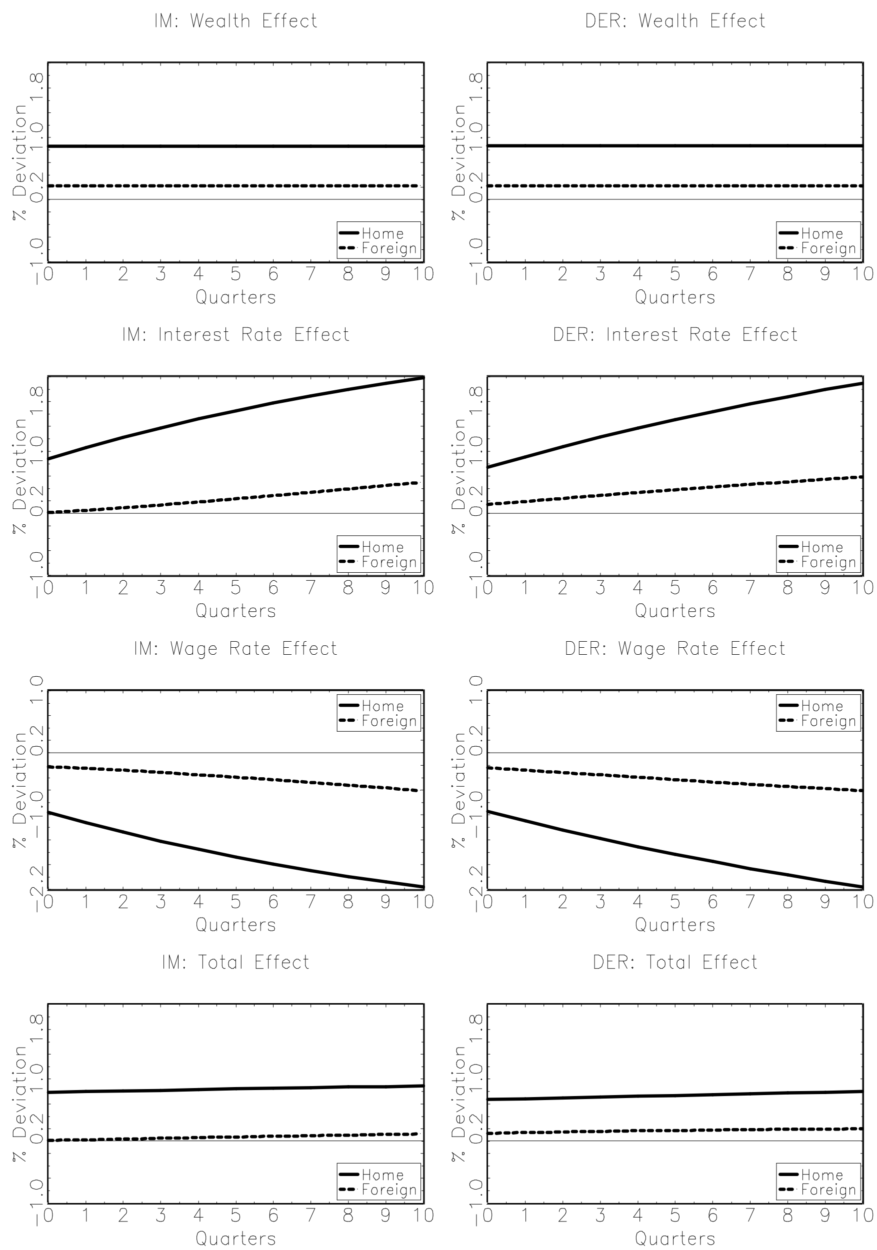
DER with BC Shock Process



Note: The figure shows the Hicksian decomposition of the responses of consumption and employment to a one standard deviation positive shock to home productivity. The responses are computed from the debt elastic interest rate model with the BC parametrization of the shock process.

Figure 6. Decomposition of Consumption Responses

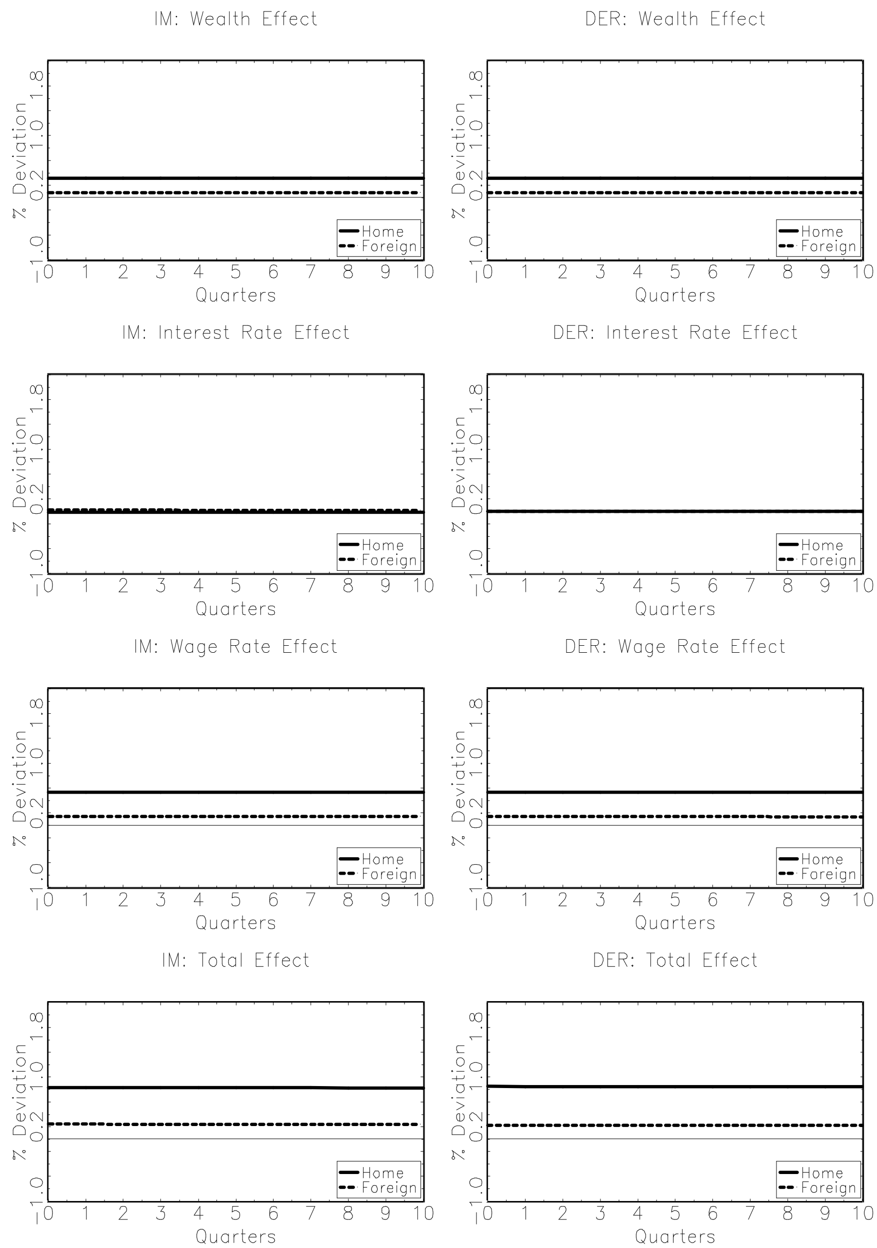
IM and DER with No N and BC Shock Process



Note: The figure shows the Hicksian decomposition of the responses of consumption to a one standard deviation positive shock to home productivity. The responses are computed from the inelastic labor versions of the baseline and debt elastic interest rate models with the BC parametrization of the shock process.

Figure 7. Decomposition of Consumption Responses

IM and DER with No K and BC Shock Process



Note: The figure shows the Hicksian decomposition of the responses of consumption to a one standard deviation positive shock to home productivity. The responses are computed from the inelastic capital versions of the baseline and debt elastic interest rate models with the BC parametrization of the shock process.