DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 09-10

Intertemporal Price Discrimination: Preference Knowledge and Capacity Choice

Samuel Raisanen

University of Colorado

November 2009

Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

Intertemporal Price Discrimination: Preference Knowledge and Capacity Choice

Samuel Raisanen[y]

Wednesday 4th November, 2009

Abstract

When a rm sells tickets in advance under capacity constraints, as with airlines, concerts, and sports tickets, one observes that in some cases advanced sales are made at a discount while other times a premium is charged. Previous research into this intertemporal price discrimination has focused on either premium or discount pricing but never both. Given that we observer both types of intertemporal price discrimination in markets characterized by advanced sales and capacity constraints, it is important to understand the conditions that determine the nature of the optimal pricing scheme. This paper is the rst to shows that the nature of the pro t maximizing intertemporal pricing scheme depends on the interaction of consumer preference intensity, preference certainty, and rm capacity. We then examine optimal capacity choice as a function of consumer preferences showing that the choice of capacity will be a negatively related to its cost for a given level of preference intensity and equal to the ex-post demand for one of the goods.

---

[*] I would like to thank my advisors Yongmin Chen, Martin Byford, and Jennifer Lamping for their many many comments and direction. Additionally, I want to extend my gratitude to the seminar participants at Otago, University for their feedback and questions.

[†] *Address*: University of Colorado at Boulder, Department of Economics 256 UCB, Boulder, CO 80309 USA, *E-mail address*:Samuel.Raisanen@colorado.edu

1 Introduction

When purchasing a ticket for a flight, typically a lower price is paid if the purchase is made well in advance of the departure date. When attending a concert or sporting event those who purchase in advance of the event pay a higher price than those who purchase the day of the show. Gale and Holmes (1992) and Gale and Holmes (1993) examined the causes for advance purchase discounts in the airlines markets. DeGrabba and Mohammed (1999) looked at the prevalence of premium pricing for bundled concert tickets. As yet, no unified setting capable of addressing the question of what conditions lead to premium pricing verses discount pricing in capacity constrained markets exists.

This paper develops a model in which a capacity constrained monopolist sells two horizontally differentiated goods in two time periods. Consumers will vary in both their preference certainty and their preference intensity. These consumer characteristics in conjunction with the firms capacity will give rise to premium, discount, and uniform pricing. Consumers who purchase early do so to ensure they receive a good which may sell out. Consumers who are uncertain as to which good they will prefer later may wait to purchase. When a firm has a low capacity relative to the market size, high prices will prevail in both purchase periods. For medium capacities, a premium can be charged to those who purchase in advance because they are willing to pay a higher price to ensure they receive their preferred good. The firm has an incentive to price below the market clearing level if it knows it can charge a high price in advance and gain on these early sales. When capacity is large, consumers who are uncertain of their future preferences can be induced to purchase early via a discount. The firm benefits by smoothing demand for the goods because those that purchase in advance do so more evenly than those who wait. If most consumers already know their preferences, low prices in both time periods will prevail.

Ex-ante, a firm may be able to choose their capacity. Knowing which pricing scheme will be profit maximizing for each level of capacity reduces the capacity choice problem to a function of consumer preference certainty. This capacity choice will be discontinuous as the choice of pricing plans is discrete. As consumers become more preference uncertain, uniform pricing and premium pricing becomes less profitable relative to discount pricing. The choice of capacity will always be that which is profit maximizing for the resulting pricing scheme and thus has discontinuities when additional preference uncertainty causes a change in the pricing plan. We will show when capacity cost is low and consumers are generally uncertain as to their future preferences, capacity sufficient to satisfy all demand under discount pricing is optimal. When some consumers know their preferences in advance and capacity is relatively costly, the profit maximizing capacity is just sufficient to meet demand for the less popular good. Finally, when capacity is very expensive, relatively low capacity should be purchased so that only the highest valuation consumers purchase.

2 The Model

Our model of intertemporal pricing uses a monopolist selling two horizontally differentiated goods in two time periods. Consumers maximize their expected surplus by choosing when to purchase. Consumers differ in both their preference certainty and preference intensity. First, the monopolist's profit maximization problem will be defined. Next, consumers' characteristics will be described. Finally, the consumers' surplus function will be made explicit. This will lead into Section 3 which uses the consumer behavior to determine what prices a firm may choose to maximize profits.

A monopolist produces and sells two horizontally differentiated goods in two time periods. The goods, good-A and good-B, can be produced at a fixed marginal cost of zero[1]. There is a common capacity of K for both goods[2]. The goods are sold in two time periods: in advance at $t = 0$ and at the day-of-consumption at $t = 1$. The monopolist maximizes profits by choosing prices for each good and at each time period, committed to advance. Prices for both goods will be the same due to the ex-ante symmetry of consumers. There is no discounting between the periods. This follows the treatment of advance purchase discounts in the Gale and Holmes (1992) model. Denote the quantity of good-g demanded at time t as Q^D_{gt} and its price as p_{gt}. Additionally, let $Q^S_{g1} = \max\{K - Q^D_{g0}, 0\}$ be the supply of good-g remaining at $t = 1$. The monopolist's profit maximization problem is then,

$$\begin{aligned}\max_{p_{A0},p_{B0},p_{A1},p_{B1}} \quad & p_{A0}\min\{Q^D_{A0}, K\} + p_{B0}\min\{Q^D_{B0}, K\} + \\ & p_{A1}\min\{Q^D_{A1}, Q^S_{A1}\} + p_{B1}\min\{Q^D_{B1}, Q^S_{B1}\}. \end{aligned} \tag{1}$$

There is a unit mass of risk-neutral consumers that vary in their preference certainty

[1] Alternatively, both prices and consumer valuations can be thought of as net with respect to a constant non-zero marginal cost.

[2] Separate capacities for each good are possible but imposed symmetry of the goods will lead to identical capacities.

and preference intensity. Preference intensity is the valuation consumers place on each good. Let be the proportion of high valuation consumers. They value their preferred good at 100 and their non-preferred good at v_{HN}. There are 1 low valuation consumers who value their preferred good at $v_{PL} < 100$ and there non-preferred good at v_{LN}. Define the expected valuation of a randomly chosen good as $_j$ for $j \in \{L, H\}$. As proven in Gale and Holmes (1992), for discounts to be profit maximizing, $_H < _L$. This will be taken as given. Each consumer has unit demand for the goods and desires at most one of either good-A or good-B. All consumption occurs at $t = 1$.

Independent of preference intensity, consumers will either be preference certain, knowing their future preferences over good-A and good-B in advance, or be preference uncertain, not knowing their future preference in advance. Let their be a proportion of preference uncertain consumers. Preference uncertain consumers, denoted type-U, are equally likely to prefer each good at $t = 0$ and learn their preference at $t = 1$. Ex-post, a proportion $> \frac{1}{2}$ of the preference uncertain consumers will prefer the peak good. The peak good is the good with the higher ex-post demand. 1 consumers will prefer the non-peak good. Ex-ante each good is equally likely to be peak. The remaining 1 consumers are preference certain. For tractability of the model, half of these consumers will prefer good-A and half will prefer good-B.[3] Denote these consumers as type-A and type-B respectively. Preference certain consumers know their preferred good at $t = 0$. At times it will be useful to refer to a consumer group by both their preference certainty and their preference intensity. A type-C_v consumer will be of preference certainty $C \in \{A, B, U\}$ and of preference intensity $v \in \{L, H\}$. Because of the presumption that these consumer characteristics are independent within the unit mass of consumers, the number of type-C_v consumers is the product of the two proportions. For example, there are $\frac{1}{2}$ type-A_H consumers.

[3] Relaxing this symmetry assumption will result the possibility that the firm may price the goods differently. This complicates the model without sufficiently changing the qualitative results.

where $p_0 = p_1$, premium pricing where $p_0 > p_1$, and discount pricing where $p_0 < p_1$. Via the lemmas in this section, we will show that there are only two potential pro t maximizing uniform prices, two potentially pro t maximizing discount pricing schemes, and a single class of premium prices.

3.1 Uniform Pricing

There are only two potentially pro t maximizing uniform pricing strategies. When capacity is low it may be optimal to charge a high price in both periods serving only high valuation consumers. When capacity is high it may be optimal to charge a low price in both periods and sell to all consumers. In both cases preference certain consumers will purchase in advance at $t = 0$ and preference uncertain consumers will wait and purchase at $t = 1$.

When capacity is su ciently low to serve only high valuation consumers, a uniform high price of $p_0 = p_1 = 100$ will be pro t maximizing.

Lemma 1. *For any given $K>0$, any pricing schedule $(p_0; p_1)$ such that $p_0 = p_1 \neq 100$ and $p_0 = p_1 > v_{PL}$ is not pro t maximizing.*

Proof. If $K = 0$, pro ts from any pricing plan are zero, so restrict to $K > 0$. If $100 > p_0 = p_1 > v_{PL}$ no low valuation consumers will purchase. All high valuation consumers will demand the good. Increasing $p_0 = p_1$ does not change demand in this range, thus increasing pro ts. So any price $100 > p_0 = p_1 > v_{PL}$ is not pro t maximizing. If $p_0 = p_1 > 100$ then no one purchases. In this case pro ts are zero and any price $p_0 = p_1 \leq 100$ yields higher pro ts because high valuation consumers will purchase. Therefore, any price $p_0 = p_1 > 100$ is not pro t maximizing. Thus, any pricing schedule $(p_0; p_1)$ such that $p_0 = p_1 \neq 100$ and $p_0 = p_1 > v_{PL}$ is not pro t maximizing. □

price may prevail. In this case, all preference certain consumers will purchase in advance at $t = 0$ and all preference uncertain consumer will wait and purchase day-of at $t = 1$.

Lemma 2. *For any given $K > 0$, any pricing schedule* $(p_0$

that will result in some consumer types purchasing in advance and others purchasing at the time of consumption.

In the case of a discount $p_0 < p_1$. The lower price in advance will induce all the preference certain consumers with valuations above p_0 to purchase in advance. In addition, this will induce all the uncertain consumers with an expected valuation below the price to purchase in advance. These uncertain consumers who purchase in advance will be evenly split between the two goods. If they had waited, of them would have purchased the peak good. Because $> \frac{1}{2}$, inducing these consumers to purchase in advance causes them to be more evenly split between the two goods. This frees up additional capacity from the peak good by shifting some of it to the non-peak good. The freed up capacity can then be sold resulting in an increase in total quantity. This potentially increases pro ts.

Lemma 3. *For any capacity $K > 0$, any pricing strategy $(p_0; p_1) \in \{(\ _L; 100); (v_{PL}; 100)\}$ such that $p_0 < p_1$ is not pro t maximizing.*

Proof. For $p_0 < p_1$, the discrete nature of the consumer space means that it is optimal to raise a price up to the point at which some set of consumers change their behavior. Thus any pricing plan such that $p_0 < p_1$ and

$$(p_0; p_1) \in \{(\ _H; \ _L); (\ _H; v_{PL}); (\ _H; 100); (\ _L; v_{PL}); (\ _L; 100); (v_{PL}; 100)\}$$

is not pro t maximizing.

For pricing schedules $(\ _H; \ _L)$ and $(\ _H; v_{PL})$, everyone except type-U_H consumers purchase in advance. In either case, the monopolist can raise p_0 to $\ _L \quad \ _H; \ _L$

$_H; v_{PL}$ is not pro t

$_H$;

this increases profits, pricing plan $(\ _H; 100)$ is not profit maximizing.

For pricing plan $(\ _L; v_{PL})$ all Type-A and Type-B consumers purchase in advance. Type-U_H consumers wait to purchase and Type-U_L consumers are indifferent. Presuming that Type-U_L consumers purchase at the lower price at $t = 0$ when they are indifferent, the monopolist can increase p_1 to 100 without changing any consumer behavior. Type-U_H consumers pay a higher price increasing profits. Pricing plan $(\ _L; v_{PL})$ is thus not profit maximizing.

Removing these non-optimal pricing plans, any pricing plan $(p_0; p_1) \in \{(\ _L; 100); (v_{PL}; 100)\}$ with $p_0 < p_1$ is not profit maximizing. $\square$

Intertemporal price discrimination can also be achieved by charging a higher price in advance and a below market clearing price at time period

Lemma 4. *For any capacity* $0 < K < \frac{1}{2}$ *and prices* $p_0 > p_1$ *such that* $(p_0; p_1)$ 2 $f(v_{PL} + \; ; v_{PL})g$ *where*

$$= \frac{\frac{1}{2} \quad K}{(1 \quad)(\frac{1}{2} \quad \frac{1}{2} \quad) + \frac{1}{2}}(100 \quad v_{PL}): \tag{3}$$

$(p_0; p_1)$ *is not pro t maximizing.*

For any $K \quad \frac{1}{2}$ *there are no prices such that* $p_0 > p_1$ *are pro t maximizing.*

Proof. If $p_0 > p_1 >$

the size of the premium as $\quad = p_0 \quad p_1$,

$$= (1 \quad R)(100 \quad v_{PL}) = \frac{\frac{1}{2} \quad K}{(1 \quad)(\frac{1}{2} \quad \frac{1}{2} \quad) + \frac{1}{2}}(100 \quad v_{PL}):$$

Finally, at $R > 1$, $p_1 < p_0$. This corresponds to the restriction that $K < \frac{1}{2}$. $\square$

Intertemporal price discrimination can only take the form of two discount pricing plans and the one premium pricing plan.

Proposition 2. *For any $K > 0$, any pricing plan*

$$(p_0; p_1) \; 2 \; f(\quad _L; 100); (v_{PL}; 100); (v_{PL}; (v_{PL} + \quad ; v_{PL})g$$

such that $p_0 \; 6\!\!= \; p_1$ is not pro t maximizing where $\quad > 0$ is the size of the premium de ned in Equation 3.

Proposition 1 and Proposition 2 restrict the potentially pro t maximizing prices to one of ve pricing plans. Because any pricing plan is either uniform, a premium, or a discount, this exhausts all potentially optimal prices.

4 Pro t Maximizing Pricing

The pro t maximizing prices can be solved by examining the pro ts from each the pricing plans from Section 3. This section divides the parameter space into regions de ned by the demands for the peak and non-peak goods under each of the ve potentially pro t maximizing pricing plans. Within each of these regions the pro ts of the relevant pricing plans are calculated and the conditions on which pricing plan is optimal is derived.

Because the regions within the parameter space where each pricing plan is optimal are complicated, the pricing space will be shown for a few speci c parameter values. The conditions that are derived for when each pricing schedule is optimal is general, the graphs are drawn for the parameters in Table 4. Half the consumers are preference uncertain. Half the consumers are of high preference intensity. Three-quarters of the preference uncertain consumers will prefer the ex-post peak good.

Parameter	Value	Meaning
v_{PH}	100	

Pricing Plan $(p_0; p_1)$	Uniform Low $(v_{PL}; v_{PL})$	Uniform High $(100; 100)$	Premium $(v_{PL} + \;; 100)$	Discount 1 $(\;_L; 100)$	Discount 2 $(v_{PL}; 100)$
Types	Purchase time				
$A_H; B_H$	0	0	0	0	0
$A_L; B_L$	0	Never	1	0	0
U_H	1	1	1	1	1
U_L	1	Never	1	0	Never

Table 2: Consumer Purchase Times by Pricing Plan

Pricing Plan	Demand Non-Peak	Demand Peak
Uniform Low	$\frac{1}{2} \quad (\quad \frac{1}{2})$	$\frac{1}{2} + (\quad \frac{1}{2})$
Uniform High	$\frac{1}{2} \quad (\quad \frac{1}{2})$	$\frac{1}{2} \quad + (\quad \frac{1}{2})$
Premium	$\frac{1}{2} \quad (\quad \frac{1}{2})$	$\frac{1}{2} + (\quad \frac{1}{2})$
Primary Discount	$\frac{1}{2} \quad (\quad \frac{1}{2})$	$\frac{1}{2} + (\quad \frac{1}{2})$
Alternative Discount	$\frac{1}{2} \quad (\frac{1}{2} \quad (1 \quad) \;)$	$\frac{1}{2} \quad (\frac{1}{2} \quad)$

Table 3: Demand

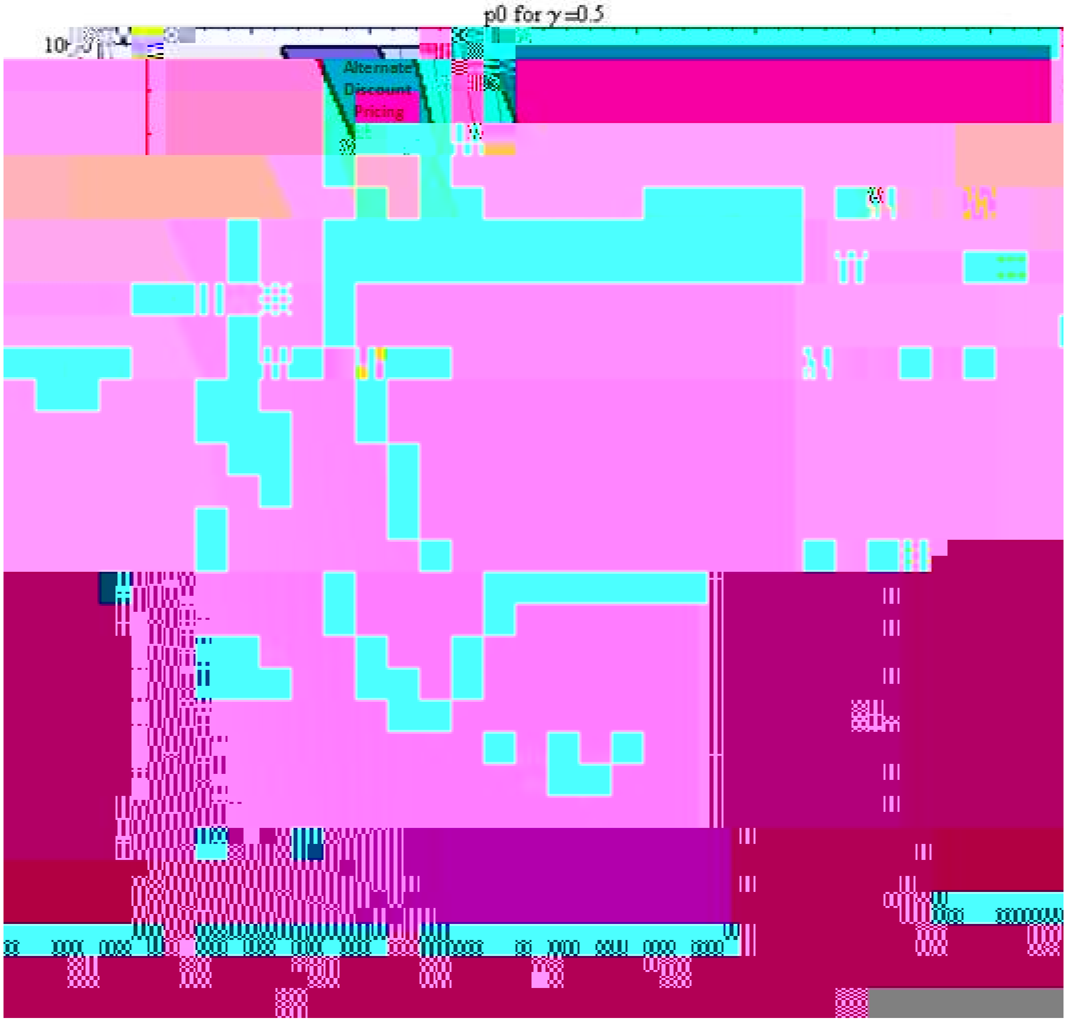

Figure 1: Pricing Regions for

of the premium that can be charged in advance is less as capacity increases because there is less rationing and a higher chance that a high valuation consumer who waits to purchase will obtain his preferred good. In net the increased sales at $t = 0$ more than offset the lower price at $t = 0$ and profits increase via premium pricing.

Now as consumers become more preference uncertain, increasing , fewer consumers are purchasing in advance as preference uncertain consumers will not pay $p_0 = 100$. When is low the additional consumers who wait to purchase do not increase the need for rationing sufficiently and thus the lower price in $t = 0$ for the Premium pricing verses the Uniform-High pricing pto4ng

Setting the advanced price premium,

primary discount pricing pro ts when

$$< \frac{100 + v_{PL} + 2 \quad 2(100 \quad v_{PL})K \quad 2 \quad _L}{2v_{PL} + 300 \quad + 2 \quad v_{PL} \quad 200 \quad 2 \quad _L}$$

Proof. For $\frac{1}{2} \quad (\quad \frac{1}{2}) \quad < K \quad \frac{1}{2} \quad (\quad \frac{1}{2})$, both goods sell out under primary discount pricing and the peak good sells out under premium pricing. Premium pro ts are then

$$_{Prem} = (v_{PL} + \quad)(\ (1 \quad)) + v_{PL}((1 \quad) \quad + (1 \quad)(1 \quad) + (K \quad \frac{1}{\quad 2})):$$

Discount pro ts are then

$$_{Disc} = \quad _L((1 \quad) + (1 \quad) \ ) + 100((1 \quad) \quad + (50 \ 50 \ 50 \ 50 \ 50 \quad _L$$

Lemma 9. *For* $\frac{1}{2} \quad (\quad \frac{1}{2}) \quad < K \quad \frac{1}{2}$*, premium pricing pro ts are higher than primary discount pricing pro ts when*

$$< \frac{100 + v_{PL} + 2 \qquad 2(100 \quad v_{PL})K \quad 2 \ _{L}}{2v_{PL} \quad + 300 \quad + 2 \qquad v_{PL} \quad 200 \qquad 2 \quad _{L}}:$$

Proof. For $\frac{1}{2} \quad (\quad \frac{1}{2}) \quad < K \quad \frac{1}{2}$, the peak good sells out under both premium and primary discount pricing. Premium pro ts are then

$$_{Prem} = (v_{PL} + \ )(\ (1 \quad)) + v_{PL}((1 \quad) \ + (1 \quad)(1 \quad) + (K \quad \frac{1}{2})):$$

Discount pro ts are then

$$_{Disc} = \quad _{L}((1 \quad) + (1 \quad) \ ) + 100((1 \quad) \quad + (K \quad \frac{1}{2} \quad \frac{1}{2}(1 \quad) \ ):$$

Solving $_{Prem} > \ _{Disc}$ yields

$$< \frac{100 + v_{PL} + 2 \qquad 2(100 \quad v_{PL})K \quad 2 \ _{L}}{2v_{PL} \quad + 300 \quad + 2 \qquad v_{PL} \quad 200 \qquad 2 \quad _{L}}:$$

□

The second change in slope happens at $K = \frac{1}{2}$. Once capacity exceeds half the market, the premium pricing degenerates into uniform-low pricing. Having more preference certain consumers no longer lowers p_1. This results in a higher tradeo between and K.

Lemma 10. *For* $\frac{1}{2} < K \quad \frac{1}{2} + (\quad \frac{1}{2}) \quad$*, uniform low pro ts are higher than primary discount pricing pro ts when*

$$< \frac{100 + v_{PL} \quad 2(100 \quad v_{PL})K \quad 2 \ _{L}}{2v_{PL} \quad + 300 \qquad v_{PL} \quad 200 \qquad 2 \quad _{L}}:$$

Proof. For $\frac{1}{2} < K \quad \frac{1}{2} + (\quad \frac{1}{2}) \quad$, the peak good sells out under both uniform low and primary discount pricing. Uniform low pro ts are then

$$_{UnifLow} = v_{PL}(\frac{1}{2} + (1 \quad) \; + K):$$

Discount pro ts are then

$$_{Disc} = \quad _{L}((1 \quad) + (1 \quad) \;) + 100((1 \quad) \quad + (K \quad \frac{1}{2} \quad \frac{1}{2}(1 \quad) \;):$$

Solving $\;_{UnifLow} > \;_{Disc}$ yields

$$< \frac{100 + v_{PL} \quad 2(100 \quad v_{PL})K \quad 2 \;_{L}}{2v_{PL} \quad + 300 \quad v_{PL} \quad 200 \quad 2 \;_{L}}:$$

$\square$

Once the capacity is large enough to sell out both the peak and the non-peak goods, increasing capacity does nothing for the discount pro ts but will increase pro ts from the uniform low pricing. The boundary then becomes increasing, leading to the telltale lower point for the discount pricing region which occurs at,

$$Q^{d}_{Peak;PrimDisc} = \frac{1}{2} + (\quad \frac{1}{2}) \quad : \tag{6}$$

Lemma 11. *For* $\frac{1}{2} + (\quad \frac{1}{2}) \quad < K \quad \frac{1}{2} + (\quad \frac{1}{2}) \;$, *uniform low pro ts are higher than primary discount pricing pro ts when*

$$< \quad v_{PL} + 2v \;_{<K}$$

pricing but neither good sells out under primary discount pricing. Uniform low pro ts are then

$$_{UnifLow} = v_{PL}(\frac{1}{2} + (1 \quad) \quad + K):$$

Discount pro ts are then

$$_{Disc} = \quad _{L}$$

Discount pro ts are then

$$_{Disc} = \quad _L((1 \qquad) + (1 \qquad)\ ) + 100 \qquad):$$

Solving $_{UnifLow} > \quad _{Disc}$ yields

$$< \frac{v_{PL} \qquad _L}{(100 \quad mu_L)}:$$

□

Finally, we must examine the alternate discount pricing region. The key to understanding high pro ts are then

$$_{UnifHigh} = 100((1 \quad) \quad + (1 \quad) \quad + K \quad \frac{1}{2} \quad):$$

Alternate discount pro ts are then

$$_{AltDisc} = v_{PL}(1 \quad) + 100(2K \quad (1 \quad)):$$

Solving $_{UnifHigh} > \ _{AltDisc}$ yields

$$< \frac{100(1 \quad K) + 50 \quad v_{PL}}{100(1 \quad) \quad 50 \quad v_{PL}}:$$

□

Once we reach the boundary between the premium pricing and high pricing region from Lemma 6, there is additional demand from the low valuation consumers. They did not demand either good under the uniform high pricing plan. This makes the premium even more attractive than high pricing was. At higher capacities this attens the slope of the border further.

Lemma 14. *For* $\frac{1}{2} \quad (\quad \frac{1}{2}) \quad < K \quad \frac{1}{2} \quad (\frac{1}{2} \quad (1 \quad) \,)$ *, premium pro ts are higher than alternate discount pricing pro ts when*

$$< \frac{v_{PL} \quad 100 \quad + 2(100 \quad v_{PL})K}{v_{PL} \quad 100}:$$

Proof. For $\frac{1}{2} \quad (\quad \frac{1}{2}) \quad < K \quad \frac{1}{2} \quad (\frac{1}{2} \quad (1 \quad) \,)$, both goods sell out under both alternate discount pricing and premium pricing. Premium pricing pro ts are then

$$_{Prem} = (v_{PL} + \quad)(\ (1 \quad)) + v_{PL}(2K \quad (1 \quad)):$$

Alternate discount profits are then

$$_{AltDisc} = v_{PL}(1 \quad) + 100(2K \quad (1 \quad)):$$

Solving $_{Prem} > \ _{AltDisc}$ yields

$$< \frac{v_{PL} \quad 100 \qquad + 2(100 \quad v_{PL})K}{v_{PL} \qquad\qquad 100}:$$

□

Once capacity is such that the alternative discount pricing will no longer sell out the non-peak good, increasing the proportion of preference uncertain consumers will reduce the capacity at which the monopolist is indifferent between premium and alternate discount pricing. This occurs when

$$Q^d_{Non \quad Peak;AltDisc} = \frac{1}{2} + (\frac{1}{2} \quad (1 \quad) \ ) \ : \tag{8}$$

This results in the sharp lower point of the alternative discount region.

Lemma 15. *For* $\frac{1}{2} \quad (\frac{1}{2} \quad (1 \quad) \ ) \ < K \quad \frac{1}{2} \quad (\quad \frac{1}{2})$ *, premium profits are higher than alternate discount pricing profits when*

$$> \ (2v_{PL} \quad 100)K + \quad + 50 \quad v$$

Alternate discount profits are then

$$_{AltDisc} = v_{PL}(1 \quad) + 100((1 \quad) \quad + K \quad \frac{1}{2}):$$

Solving $_{Prem} > \; _{AltDisc}$ yields

$$> \frac{(2v_{PL} \quad 100)K + \quad + 50 \quad v_{PL}}{100(1 \quad) + \quad vPL}:$$

$\square$

If we increase the number of preference uncertain consumers when the capacity is large enough that the non-peak good fails to sell out, the discount pricing loses consumers relative to the premium pricing. This is because all preference certain consumers purchase while only the high valuation preference certain consumers will purchase. To offset this relative effect, capacity must be lowered toward the non-peak sell out line. This creates an upward sloping border to the right of this line and thus a sharply pointed region where the alternative discount pricing is profit maximizing.

Figure 4 shows each of the relevant lines for the maximum capacity needed. These lines, as defined in equations 5 through 8, move with changing parameter values and define the optimal pricing regions. Understanding when the peak and non-peak goods sell out is critical to understanding why each pricing plan is optimal where it is. Once the non-peak good fails to sell out there is an incentive consider discount pricing to begin demand shifting to increase profits. In general, when capacity is low, profits are maximized via uniform high pricing. Once capacity is large enough to exceed high valuation demand premium pricing will most consumers are preference certain, insufficient demand smoothing occurs under discount pricing and uniform low pricing will prevail.

5 Capacity Choice

Until now capacity has been treated as an exogenous variable. Firms, however, choose their capacity. Once capacity is chosen, a pricing plan for the goods is selected. Capacity choice is thus an exercise in backward induction. The choice of capacity will then depend on the cost of capacity, consumers' preference certainty and consumers' preference intensity. Once we understand how firms choose their capacity, we will be able to understand which pricing plans will be observed in markets with differing costs of capacity. This section develops the firm's problem after incorporating the cost of capacity and then examines the relationship between capacity cost and the profit maximizing pricing plan.

Here it will be assumed that capacity can be chosen separately for both good-A and good-B, K_A and K_B respectively. This is done to match with a model of airlines where different flights have different capacities. If capacity is ex-ante required to be the same for both goods, for example a concert hall or sports arena, the $K_A = K_B$ can be assumed prior to the maximization problem without altering the results. We first show that the profit maximizing capacity must be on the demand line for either the peak or non-peak good for the optimal ex-post pricing plan. Then we examine the optimal pricing plan that results from a given marginal cost of capacity. As cost of capacity increases, the firm uses less capacity. This will result a movement away from discount and low pricing toward premium and high pricing.

plan:

$$\max_{(p_0,p_1,K_A,K_B)} p_0(\min\{Q^D_{A0}, K_A\}+\min\{Q^D_{B0}, K_B\})+p_1(\min\{Q^D_{A1}, Q^S_{A1}\}+\min\{Q^D_{B1}, Q^S_{B1}\}) - c(K_A, K_B). \tag{9}$$

We assume that the marginal cost of capacity for each good is symmetric. This along with the symmetry of consumer preferences over good-A and good-B results in the profit maximizing capacities being identical. The choice of capacities is then just a choice of a single K. Denote the marginal cost of an additional unit of capacity for both goods as $mc_K(K) = \frac{\partial c(K,K)}{\partial K_A} + \frac{\partial c(K,K)}{\partial K_B} = 2\frac{\partial c}{\partial K_A}$. For tractability mc_K is assumed to be constant. The optimal choice of capacity the depends on the marginal cost of capacity, consumers' preference intensity, consumers' preference certainty, and the ex-post peak demand pait89y then it is optimal for the monopolist to increase capacity to meet demand for either the peak or non-peak good. If $2p_0 \quad c\,(K$

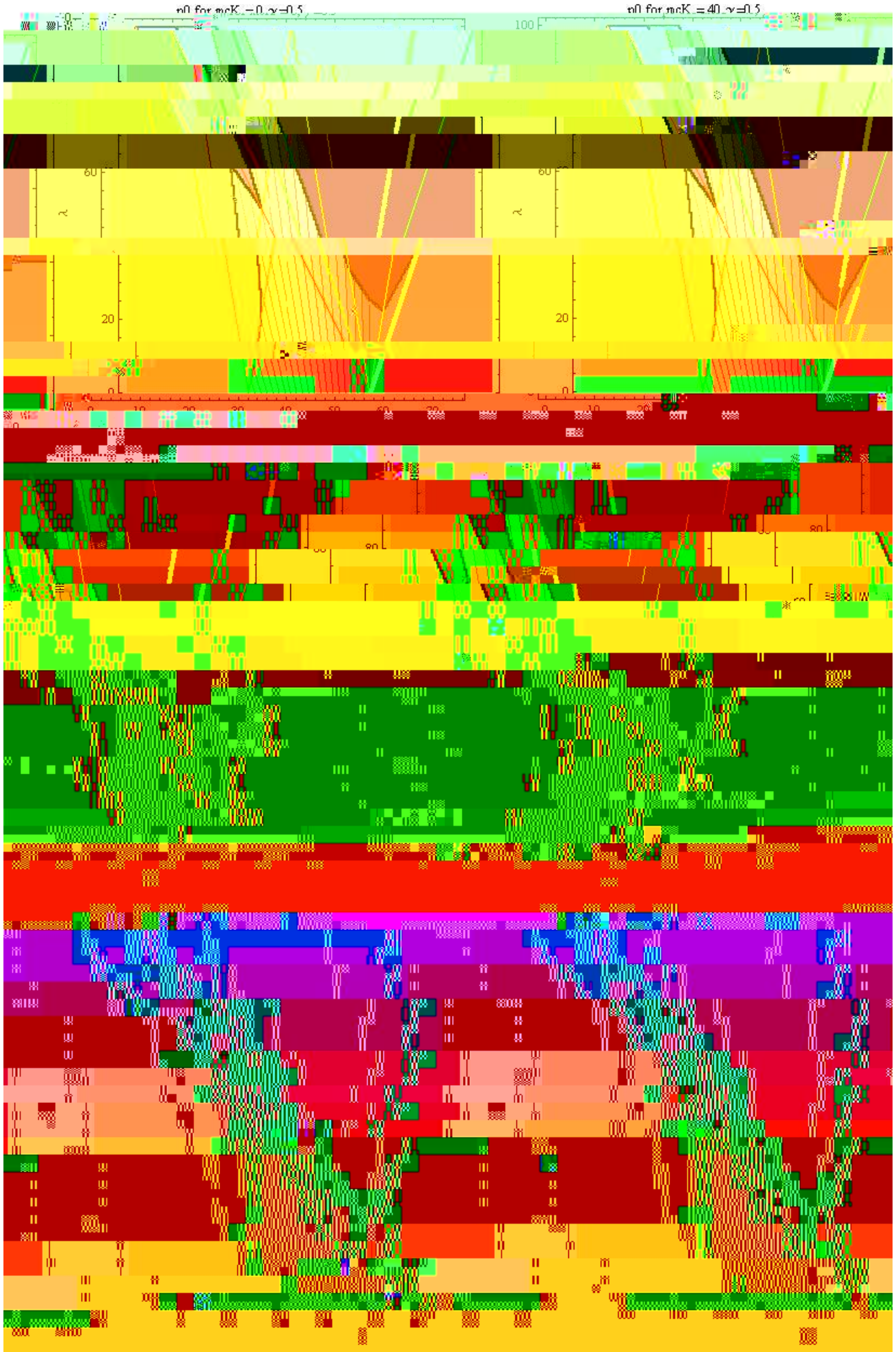

Figure 3: Pro t Maximizing Capacity (in White) for Monopolist with Demand Lines

the demand lines given in Table 4. As shown in Lemma 16, the optimal capacity does follow the demand lines.[5] For low marginal cost of capacity, optimal capacity will be large enough to sell to all consumers. The rm will then choose uniform low pricing when most consumers are preference certain and discount pricing otherwise. As the cost of capacity rises, for low amounts of preference uncertain consumers, it becomes optimal to charge a premium and chose a capacity that just sells out non-peak good. At high proportions of preference uncertain consumers the discount will still be optimal. As the cost of capacity rises above v_{PH}, it is no longer pro table to only sell one more unit of the peak good. In order to recover the cost of the capacity, the monopolist must sell both the peak and non-peak goods. As a result, the optimal capacity must be on the non-peak demand lines. Finally, as capacity becomes very expensive, it is optimal to charge a uniform high price and keep capacity low.

6 Applications and Conclusions

Thus far we have shown how premiums, discounts, and uniform pricing arise in markets with constrained capacity, preferences uncertain consumers, and multiple sales periods. A rm that charges a premium to purchase in advance takes advantage of high valuation consumers who are concerned that the good may be rationed. A rm which o ers a discount for advanced purchases does so to induce low valuation consumers to purchase before they know their preferences. This results in demand shifting away from the ex-post more popular good and increasing the overall quantity of goods sold.

From these observations we can deduce that industries for which advanced purchase discounts prevail will be characterized by many consumers who do not know their preferences well in advance. Airlines t this description. Consumers often choose between very similar ight times and may not know until closer to the departure date which time they prefer.

[5] And deviation is just due to the discreteness of the problem.

when additional capacity costs are high.

The model developed here was designed to unify the discount pricing found in Gale and Holmes (1992) and the premium pricing found in DeGrabba and Mohammed (1999). Both forms of intertemporal price discrimination can depend on the level of capacity, consumer preference certainty, and consumer preference intensity. We have shown that markets which are similar in terms being capacity constrained and sell their tickets intertemporally depend on consumer preference certainty and intensity for their pricing strategies.

To be more applicable to the applications of airline pricing and event ticket sales this model can be placed into a competitive framework. Using the intuition developed by Dana (1992), advanced purchase discounts would still prevail due to the demand shifting process. The ability of firms to charge premiums will disappear as market power erodes the ability of firms to charge above marginal cost. Introducing additional degrees of product differentiation between goods sold by different firm would allow for premiums to persist. This is left for future work. For now, it is clear that both discounts and premiums can arise in markets where a monopolist chooses capacity and then sets prices in multiple time periods. This choice is determined by the intensity of consumer preferences, the uncertainty of consumer preferences, and the cost of additional capacity.

References

James D. Jr. Dana. Advanced-purchase discounts and price discrimination in competitive markets. *The Journal of Political Economy*

Shu-Fen Yu. Price perception of online airline ticket shoppers. *The Journal of Air Transportation Management*, 14:66{69, 2008.